Time capsule for geotechnical risk and reliability

Marcin Chwała[1], Kok-Kwang Phoon[2], Marco Uzielli[3], Jie Zhang[4], Limin Zhang[5], and Jianye Ching*[6]

Abstract

This paper is motivated by the time capsule project (TCP) of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The historical developments of geotechnical risk and reliability primarily for soil mechanics was covered over the past six or more decades (1960 – 2010+). Key milestones include application of probability to quantify uncertainties and compute probability of failure, spatial variability, random field theory, first-order reliability method, random finite element method, reliability-based design, load and resistance factor design, Bayesian updating, soil and load test databases, and machine learning methods. Given the complexity of natural ground, engineering judgment remains important to bridge the gap between theory and reality. However, the role of engineering judgment needs to be updated as modern machine learning methods become more powerful.

Keywords: geotechnical risk and reliability; time capsule; history.

Introduction

Geotechnical engineering can be distinguished from other civil engineering branches by the need to grapple with very large uncertainties in material parameters and possibly unknown unknowns (e.g., geologic "surprises") (Phoon 2017). For geotechnical engineers, the most important material is natural soil. Therefore, they must constantly struggle with the spatial variability of the substrate, the uncertainty of the stratigraphy, the possibility of the presence of weak soil lenses, and other geotechnical-related uncertainties. Uncertainty quantification in geotechnical engineering is an interdisciplinary issue. It combines, for instance, probability theory (or other uncertainty quantification theory), soil mechanics, natural hazards, geology, and social issues. It is now generally accepted to perform the risk analysis associated with the subsoil for each significant structure. Some past failures clearly show the need for geotechnical engineers to manage uncertainty more rationally. Subsoil-related uncertainties motivated engineers and scientists to describe these effects by using probabilistic approaches and incorporating them into existing deterministic models or creating new approaches. These efforts are briefly described below in a short history of uncertainty quantification in geotechnical engineering. A historical review covering so many diverse topics within a single paper is necessarily incomplete and not representative of the volume of important research conducted over the past 6 or more decades (1960 – 2010+). The spirit of this paper is to take the reader on a journey akin to a wine tasting tour, where the sampled wines are the ideas that have contributed to education, research, and

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[1] Assistant Professor, Wrocław University of Science and Technology, Wrocław, Poland.

[2] Professor, Singapore University of Technology and Design, 8 Somapah Road, #07-301 Building 3, Level 7, Singapore.

[3] Assistant Professor, University of Florence, Florence, Italy.

[4] Professor, Tongji University, Shanghai, China.

[5] Professor, Hong Kong University of Science and Technology, Hong Kong SAR, China.

[6] (Corresponding author) Professor, National Taiwan University, Taipei, Taiwan. Email: jyching@gmail.com. Phone: 886-2-33664328. Fax: 886-2-23631558.

practice. In addition, there is a large literature on risk and reliability in rock mechanics, rock engineering, tunnelling, mining engineering, and hydrogeology that is not covered in this tour. Some later examples include Einstein (1991), Priest and Hudson (1981), Hoek (1999), Contreras and Brown (2018), Kitanidis (1997), and Brown (2012), but ealier papers on statistics and reliability have been published in the US and international rock mechanics conferences in the 70s. The application of risk-based design of high rock slopes in South African and Australian mining engineering activities is also noteworthy. Einstein (2003) reviewed how uncertainty has been dealt with over the past 40 years in rock mechanics using the decision making cycle as a frame of reference. Hadjigeorgiou (2019) discussed how different risk analysis tools and procedures can be used effectively in geomechanical mining. Finally, structural reliability is a more mature field with origins traced to Freudenthal (1947) and Pugsley (1955). It is outside the scope of this review as well.

In the geotechnics community, interests in probabilistic and related risk topics likely emerged in the sixties. During the Terzaghi Lecture in 1964, Casagrande (1965) presented the study titled: “Role of the ‘Calculated Risk’ in Earthwork and Foundation Engineering” where the need for risk assessment in geotechnics was indicated. Shortly thereafter, Lumb’s classical Canadian Geotechnical Journal paper on “The Variability of Natural Soils” was published and became one of the first to characterize spatial variability statistically (Lumb 1966; Phoon 2020). Wu is also regarded as one of the seminal figures who contributed to the development of geotechnical reliability (Baecher and Christian 2019). In 1965, Hansen (1956,1965) proposed the selection of partial factors based on two guidelines: (a) a larger partial factor should be assigned to a more uncertain quantity, and (b) the partial factors should result in approximately the same design dimensions as that obtained from traditional practice. It should be noted that the partial factor is applied as a divisor on the nominal soil parameter, probably a matter of convention established by the factor of safety. For the load and resistance factor design (LRFD) format adopted in North America, the resistance factor is a multiplier. Although Hansen’s guidelines were qualitative, this is the first time a link between uncertainty in the material parameter and a partial factor that influences design was proposed. Brinch Hansen was the first to use “limit state design” in geotechnical engineering. The term “limit state design” refers to a design philosophy or a design method for ensuring safety. This limit state design method can be non-probabilistic (partial factors calibrated to produce designs comparable to those produced by the global factor of safety) or probabilistic (simplified reliability-based design) (Phoon et al. 2003a). Christian and Baecher (2015) pointed out that Taylor (1948) had discussed the use of partial factors for cohesion and friction angle to account for the different uncertainties underlying the estimation of these parameters, although load partial factors and limit states were not considered. There are also other scholars that laid the foundation for different aspects of geotechnical risk and reliability in the sixties as outlined in the next section.

Casagrande (1965) defined the term “calculated risk” as:

“*a) The use of imperfect knowledge, guided by judgment and experience, to estimate the probable ranges for all pertinent quantities that enter into the solution of the problem.*

b) The decision on an appropriate margin of safety, or degree of risk, taking into consideration economic factors and the magnitude of losses that would result from failure.”

He also introduced the terms “unknown risk” and “human risk” and reviewed several case histories. Casagrande tried to demonstrate the importance of risks in earthwork and foundation engineering. Although the paper by Casagrande served as a turning point in risk assessment in geotechnics, some papers dealing with this subject were written in non-English languages. Probably this is the reason why they are not known to the general public. Therefore, before we move on to the main description of the achievements of our scientific discipline, let us mention a few pioneering papers. Among them, we can distinguish three studies related to the safety of geotechnical structures: Wastlung (1940) written in Swedish, Levi (1958) written in French, and Biernatowski (1966a,b) written in Polish. Hansen (1956) was written in Danish.

The development of uncertainty quantification in geotechnical engineering is a continuous process, but sometimes it is drastically accelerated by innovative approaches and new techniques. The work of Casagrande, Hansen, Lumb, Wu, and others in the sixties can be considered as an approximate origin where the discipline of uncertainty quantification in geotechnical engineering began to take shape. The expansion of this discipline due to this continuous process cannot be divided into separate periods; however, to bring some structure to our story, we adopted the convention of distinguishing each decade, and our effort was focused on identifying the most important achievements and ideas in those decades. A rather illuminating example of how the discipline developed is shown in Fig. 1, where the number of published papers mentioning the given words in Google Scholar is compared. Fig. 1a shows a growing interest in geotechnical-related uncertainty quantification, which is a response to the infrastructure needs of the modern world. Moreover, an increasing proportion of studies related to uncertainty issues to general geotechnical studies is observed, as shown in Fig. 1b.

Returning to our story, we start with the first steps taken by geotechnical engineers to grapple with diverse and significant uncertainties pertaining to the subsurface environment with focus on soils. The description of the pioneering decade begins.

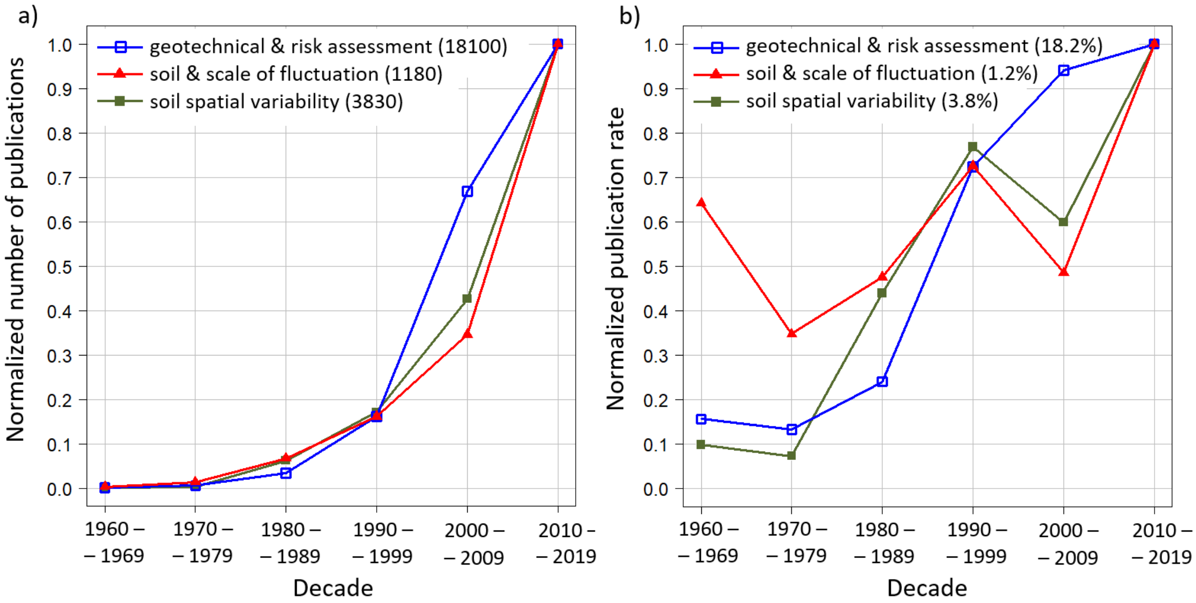

Fig. 1. Normalized number of publications containing specific phrases; (a) normalization is with respect to total number in the 2010-2019 decade shown in parentheses); (b) normalization using the ratio of the number of publications containing specific phrase to the number of publications containing the phrase "geotechnical engineering" (percentage in the parenthesis refers to ratio in the 2010-2019 decade).

1960-1970

The study that deserves to be mentioned first is the pioneering Lumb (1966) paper. He titled his study "The variability of natural soils" and it is considered to be the starting point for a new branch of geotechnics (Phoon 2020). Lumb showed that the variations in properties for natural soils can be described by random variations about a mean or linear trend, related to a probability distribution. As a basis, he considered four typical Hong Kong soils – a soft marine clay, an alluvial sandy clay, residual silty sand, and residual clayey silt. The studied properties include Atterberg limits, grading, and, for undisturbed samples, strength and compressibility characteristics. He provided examples of soil properties following the normal, lognormal, and binormal distributions. A relatively large sample size was an advantage of Lumb's study. Moreover, Lumb stated that the probability the parameter could be less than the design value is a rational basis for the choice of design parameters. It was the first study to guide later research on application of probability to quantify geotechnical parametric uncertainties numerically. For example, Eurocode 7 (Comité Européen de Normalisation 2004), Section 2.4.5.2 Clause 11 states "If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%." This fractile definition of the characteristic value can be found in Lumb (1966). Details are given elsewhere (Orr 2015). At the same time as Lumb's classical paper was published, other studies concerning the application of probability theory to engineering structures began to appear. The paper by Langejan (1965) discussed the slope stability problem. Biernatowski (1968) also considered probabilistic solutions for slope stability. One year earlier a pioneering paper by Wu and Kraft (1967) was published. The authors used the probability of failure to quantify the safety of foundations for several load and strength distributions. The applied load and soil strength were considered to be random variables. They determined the appropriate probability function (a normal distribution was used) by fitting the experimental data. Kraft's dissertation (Kraft 1968) and related papers (Wu and Kraft, 1967; 1970a; 1970b) were some of the first probabilistic assessments of foundation safety for bearing capacity and settlement. Wu (1974) summarized and extended this body of work in his 1974 paper on "Uncertainty, Safety, and Decision in Soil Engineering". In principle, measurement error can be determined directly by analyzing the variation of the results obtained by a representative group of soil testing companies performing the "same" test on nominally identical soil samples. Comparative testing programs of this type were conducted by Hammitt (1966) and Johnston (1969). The above-described studies laid the foundations for the wider use of probabilistic methods to describe uncertainties in geotechnics, which was revealed in the next decade.

1970-1980

The initial steps in describing uncertainties in geotechnical engineering in the sixties ushered in a period of tremendous progress in the seventies. There are a few reasons for this. One of them is Lumb's initiation of a conference series dedicated to probability theory applications in geotechnics, i.e., the International Conference on Applications of Statistics and Probability to Soil and Structural Engineering (ICASP). This conference series promotes a probabilistic viewpoint on soil parameters among other topics. In the 1970s, three ICASP conferences took place, Hong Kong in 1971, Aachen in 1975, and Sydney in 1979. Another reason for the increasing popularity of the probabilistic approach for uncertainty description is the development of appropriate computational methods, e.g., calculation of the first-order reliability index by Hasofer and Lind (1974) and an algorithm for the calculation of structural reliability under combined loadings by Rackwitz and Flessler (1978). In the seventies, Lumb continued to investigate probabilistic descriptions of soil parameters, e.g., in his 1970 paper (Lumb 1970), he compared the natural variabilities of cohesive and frictional components of strength and found that the beta probability distribution agrees more closely with experimental data than the commonly assumed normal distribution. Lumb also authored ‘Chapter 3: Application of Statistics in Soil Mechanics’ in the book “Soil Mechanics – New Horizons” (Lumb 1974). After Lumb, studies by other researchers aiming at a probabilistic description of subsoil parameters started to appear. Soil parameters as random variables were investigated by Schultze (1972, 1975). An extensive study for multiple samples was conducted by Corotis et al. (1975). The authors showed the results in three groups based on the bulk density of soil. They found that the probability characteristics depend on the bulk density of the soil and therefore on its type and that most of the variations can be described by either normal or lognormal distribution. The statistical characterization of soil properties remained an important topic of interest (e.g., Singh and Lee 1970; Kay and Krizek 1971; Haldar and Tang 1979a). Apart from describing uncertainties in soil properties, more applications to geotechnical structures became accessible in the seventies. Among important studies of that time, one can find a study by Singh (1972), where the investigation on how reliable is the factor of safety in foundation engineering is performed. The author investigated retaining structures, bearing capacity problems, and slope stability analyses. The stability of rigid structures in the probabilistic formulation was also considered by Biernatowski (1972). In 1976, a classic paper on slope stability analysis was published by Alonso (1976). Alonso tried to define better slope safety measures by using a probabilistic approach. He used a mechanistic description of stability (the method of slices) for Canadian sensitive clays and implemented first-order probability analysis to allow rational evaluation of the different sources of uncertainty. He found that the uncertainties in the cohesion parameter, the pore-pressure, and the method of analysis are the relevant ones governing the uncertainty in the estimation of slope safety. Probability-based short-term and long-term designs of soil slope were investigated by Tang et al. (1976) and Yuceman and Tang (1975), respectively. Wilson H. Tang contributed significantly to the reliability analysis of slopes, design of offshore foundations, and the promotion of the use of Bayesian methods in geotechnical engineering (Lacasse et al. 2017). His lifetime contribution to the field of geotechnical reliability was summarized in ASCE GSP 286 (Juang et al. 2017) and Proceedings of the Professor Wilson Tang Memorial Symposium (Zhang et al. 2012). de Mello (1977) is the first Rankine lecturer to discuss the application of statistics and probability to decision making in dam engineering. He expanded on the central idea in his General Report at the second ICASP in Aachen 1975 that the statistics of averages is distinct from the statistics of extreme values (de Mello 1975). This distinction is now codified in Eurocode 7 (Comité Européen de Normalisation 2004), Section 2.4.5.2 “Characteristic values of geotechnical parameters”, Clauses 7 and 8. Clause 7 states that the “zone of ground governing the behaviour of a geotechnical structure at a limit state is usually much larger than a test sample or the zone of ground affected in an in situ test. Consequently the value of the governing parameter is often the mean of a range of values covering a large surface or volume of the ground. The characteristic value should be a cautious estimate of this mean value.” The statistics of averages is thought to apply to this “large surface or volume of the ground.” A specific example of slope stability was highlighted by de Mello (1977). However, it is now understood that the statistics of averages and the statistics of extreme values are both needed in the presence of spatially variable soils, because critical slip curves seek the weakest kinematically admissible paths by definition (Ching and Phoon 2013a, 2013b; Ching et al. 2014, 2016, 2017).

The statistics of spatial averages in spatially variable soils was first studied formally by Vanmarcke using random field theory (Vanmarcke 1977a, 1977b). Vanmarcke stated that when the degree of disorder is sufficiently large, there is usually merit and economy in probabilistic rather than deterministic models. He pointed out that random field theory seeks to model complex patterns of variation (interdependence, correlation) in cases where deterministic treatment is inefficient and conventional statistics are insufficient. Vanmarcke stated that central to the development of robust random field models is the concept of the “local average” of a random field that is more relevant to geotechnical engineering practice than point variation. Vanmarcke’s “local average” is different from the “statistics of averages” of de Mello (1977) in key aspect. He considered spatial variability or spatially correlated soil properties, which is visibly present in all soil profiles. Vanmarcke further stated that the ideal random field model should capture the essential features of a complex random phenomenon by a minimal number of physically meaningful and accessible parameters. His studies began a new branch of soil parameters description by using random fields and were widely used in the following decades.

The probability approach has been also incorporated into dynamics issues, e.g., Haldar and Tang (1979b) proposed a procedure to estimate the probability of liquefaction for a given design earthquake magnitude and acceleration, initiating the research on the application of probabilistic methods in soil liquefaction problems. At the end of the 1970s, the first geotechnical applications of the stochastic finite element method (FEM) appeared, e.g., stochastic analysis of steady-state groundwater flow in a bounded domain by Smith and Freeze (1979a, b). However, the finite element-based approaches found wider use in the next decade. The increasing power and decreasing cost of computing machines should explain this rising popularity in later years. Phoon et al. (2022a) opined that the increasing power and convergence of digital technologies beyond computing machines will usher the next wave of change in research and practice.

The amount of uncertainty involved in a geotechnical design can be reduced through additional information. Such an uncertainty reduction process can be formally formulated based on Bayes’ theorem. In the 1970s, the potential of uncertainty updating through Bayes’ theorem had been recognized by Wilson H. Tang. It is worth highlighting that Tang (1971) introduced how to quantify the value of additional data in geotechnical engineering through Bayesian analysis. Tang (1973) systematically illustrated the concept of how the Bayesian method can potentially be used for uncertainty reduction in geotechnical engineering. Kay (1976) suggested a Bayesian method to reduce the uncertainty associated with the bearing capacity of pile foundations through load test data, which can then be used to improve pile design using reliability theory.

In 1970, Benjamin and Cornell (1970) published the first textbook which systematically introduces the knowledge of probability and statistics for students, practitioners, teachers, and researchers in civil engineering, and was inspiring for early researchers in geotechnical engineering. The classic textbooks by Ang and Tang (1975, 1985), which provided comprehensive examples on how probabilistic methods can help improve civil engineering analysis, made the concepts and methods of reliability analysis much more accessible to the geotechnical profession. These textbooks were revised in 2007 (Ang and Tang 2007), and have been translated into 5 languages and used worldwide.

As early as the 1970s, researchers at the Norwegian Geotechnical Institute (NGI) started to contribute to geotechnical risk and reliability. Folayan et al. (1970) developed a Bayesian updating method to predict the settlements of a marshland development and analyzed the associated economic consequences. Høeg and Murarka (1974) adopted a probabilistic approach to calculate the probability of failure of a retaining wall. A number of these ideas originated from the PhD studies of Folayan and Murarka conducted at Stanford University.

This is also a period of overlap between geotechnical reliability and the then more mature field of structural reliability (Cornell, Veneziano, Rackwitz, Madsen, Schuëller, Corotis, Hasofer and Lind, Ditlevsen, Thoft-Christensen, Melchers, etc.). The literature on structural reliability is extensive. For the past five decades, twelve International Conference on Structural Safety and Reliability (ICOSSAR) have been organized by the International Association for Structural Safety and Reliability (IASSAR). The first ICOSSAR was held in 1969 in Washington, D.C., USA. Since 1977, it has been successfully held every four years at venues in Europe, USA and Japan: Washington, D.C., USA (1969), Munich, Germany (1977), Trondheim, Norway (1981), Kobe, Japan (1985), San Francisco, USA (1989), Innsbruck, Austria (1993), Kyoto, Japan (1997), Newport Beach, USA (2001), Rome, Italy (2005), Osaka, Japan (2009), New York, USA (2013), Vienna, Austria (2017) and Shanghai, China (2022).

1980-1990

The growing popularity of probabilistic methods in geotechnics enhanced by new computational methods, e.g., Ditlevsen (1981a) and Hohenbichler and Rackwitz (1981), resulted in increased activity by researchers in this field. There were two ICASP conferences in the 1980s decade, Florence in 1983 and Vancouver in 1987. The ASCE proceedings "Probabilistic Characterization of Soil Properties: Bridge between Theory and Practice" published in 1984 is noteworthy (Bowles and Ko 1984). In 1983, a classic book by Vanmarcke was published, "Random Fields – Analysis and Synthesis" (Vanmarcke 1983). Tang (1984) proposed a simplified method to estimate spatial-averaged soil properties. In 1983, McAnally (1983) investigated the reliability of the bearing capacity designs of shallow footings in sands and proposed a procedure that allows foundations to be designed with a more consistent level of reliability than those obtained by using a single value of safety factor. Biernatowski and Puła (1988) proposed an approach for probabilistic analysis of the stability of massive bridge abutments. In 1988, an original approach of Kandaurov's stochastic model of granular soil was developed by Brząkała (1988). However, the greatest interest was in the slope stability analysis, e.g., Li and Lumb (1987) discussed some improvements on the first-order second-moment (FOSM) probabilistic approach to slope design, which had originally been introduced by Cornell (1971) at the first ICASP conference in Hong Kong. Ditlevsen (1981b) extended Cornell's FOSM to large systems, which is important for real-world problems. Chowdhury et al. (1987) investigated the progressive development of slope failure within a probabilistic framework. Ishii and Suzuki (1986) proposed the stochastic finite element method (FEM) that uses the first-order approximation at a failure point of a set of random variables. Apart from the studies on slope stability and foundation bearing capacity, soil liquefaction was also investigated, e.g., a consistent set of stochastic models was developed by Fardis and Veneziano (1981) for the liquefaction resistance of a homogenous mass of sand. Baecher et al. (1980) conducted one of the first studies on the risk assessment of dams. Einstein and Baecher (1982) systematically introduced how probabilistic and statistical methods can be applied in the field of engineering geology. A more comprehensive treatment is presented by Hartford and Baecher (2004). The state-of-the-art is reviewed by Baecher (2016).

The development of probabilistic methods in geotechnics was also motivated at that time by the need of designing drilling platforms, offshore structures, and their foundations as experiences on the analysis, design, and construction of such structures are rare. Wu et al. (1989) provided a state of the art review on the reliability of offshore foundations. The need for proper soil spatial variability characterization became important for marine soils, e.g., Wu et al. (1987). Kraft joined the offshore oil business after his PhD at the Ohio State University and continued to publish seminal papers in this field (e.g., Kraft and Murff 1976). Initiated by these pioneering studies, offshore foundations have become one of the areas where geotechnical reliability method has found successful applications.

Robert V. Whitman in his Terzaghi lecture in 1981 opined that "probability theory is regarded with doubt and even suspicion by the majority of geotechnical engineers" (Whitman 1984), and he mentioned the language barrier as one of the possible reasons for this situation. However, fortunately, this decade can be considered as a turning point in the attitude of geotechnical engineers to probabilistic methods. This is mainly due to good examples of how these methods have been applied in practice. Important studies that investigated real geotechnical structures from a probabilistic point of view were published in the 1980s. Among them was a study by Duncan and Huston (1983) on estimating the probability of failure of California Delta levees by using simple statistical procedures. Their analyses were based heavily on empirical data. An interesting study of probabilistic analysis in the assessment of dam safety issues was published by Vick and Bromwell (1989). Their examples showed that probabilistic methods were expanding into geotechnical engineering practice. It is worthwhile pointing out Baecher's contribution to making geotechnical risk and reliability more accessible to non-specialists. Examples include the FHWA "Geotechnical Risk Analysis User's Guide" (Baecher 1987), and two special MIT summer courses, "Geotechnical Error Analysis" (Baecher 1985) and "Reliability Analysis of Stability of Embankments of Soft Clays" (Baecher and Ladd 1985).

The 1980s also saw a significant development in the applications of the stochastic FEM in geotechnics. Baecher and Ingra (1981) used the stochastic FEM to predict uncertainties in total and differential settlements under a large flexible footing. Righetti and Harrop-Williams (1988) modelled a structure by a limited number of accessible stochastic data and computed the characteristics of the displacement and stress random fields in the structure by the first-order second-moment approximation. The authors considered the application of stochastic FEM for a soil profile with a random distribution of the elastic modulus.

In the 1980s, the benefits of Bayesian methods in geotechnical engineering were further explored. Notably, Baecher and Rackwitz (1982) suggested that the variability of the bearing capacity can be divided into within-site and cross-site variabilities and illustrated how the cross-site variability can be reduced through the Bayesian method. The above concept later initiated the development and application of hierarchical Bayesian models for geotechnical engineering applications in the 2020s (e.g., Zhang et al. 2014, 2016; Bozorgzadeh and Bathurst 2020; Ching et al. 2021a, 2021b; Xiao et al. 2021). Tang and his co-works developed Bayesian methods for detecting anomalies through site investigation (e.g., Tang and Quek 1986; Tang 1987; Tang et al. 1988; Tang and Halim 1988).

As one can observe, the 1980s brought tremendous development of probabilistic approaches in geotechnical engineering. For completeness, it is worthwhile to point out the work of Harr (1977, 1987), which is somewhat outside the mainstream development but nonetheless influential on the U.S. Army Corps of Engineers through the students he supervised at Purdue University and who later went on to work there, especially Wolff (1985, 1994, 1995). A more complete summary of this body of work is given elsewhere (Wolff 2008). As a derivative of this, the next decade brought a wider application of probabilistic methods to geotechnical engineering.

1990-2000

An important aspect influencing the development of probabilistic methods in geotechnics in the 1990s was the growing computing power of computers (especially personal computers). This made it possible, for example, to use a combination of the Monte Carlo method and the finite element method (named random FEM, or RFEM). Such an approach was intensively developed after the publication of the first studies by Fenton and Griffiths. They investigated seepage beneath water retaining structures founded on spatially random soil (Griffiths and Fenton 1993) and estimated the distribution of an equivalent conductivity measure, the block conductivity, which characterizes the total flow rate through a two-dimensional bounded domain and which is itself a random variable (Fenton and Griffiths 1993). In the paper by Paice et al. (1996), RFEM was used for settlement modeling on spatially random soil. Stochastic FEM was further developed in the 1990s (e.g., Spanos and Ghanem, 1989) and used for other geotechnical applications, e.g., analysis of soil layers with random interfaces (Ghanem and Brząkała, 1996). Simpler perturbation-based stochastic FEM was explored by Phoon et al. (1990) and Quek et al. (1991, 1992). Foundation settlements for layered soil were also studied using stochastic FEM by Brząkała and Puła (1996). In the 1990s, ICASP conferences continued and provided a platform for the exchange of ideas in the geotechnical research community, in this decade three ICASP conferences took place, i.e., Mexico City in 1991, Paris in 1995, and Sydney in 1999.

The increasing interest in using the random field for soil spatial variability description resulted in a growing need for random field properties estimation and simulation. In response many important studies on soil parameter variability were published in the 1990s, e.g., Fenton and Vanmarcke (1990), Kulhawy et al. (1991), Jaksa (1995), Lacasse and Lamballerie (1995), Lacasse and Nadim (1996a), and Fenton (1999a, b). In 1999, Phoon and Kulhawy (1999a, b) decomposed uncertainties into spatial variability, measurement error, statistical uncertainty, and transformation uncertainty, which was of great influence on how we currently model uncertainties. Jaksa et al. (1999) investigated experimentally vertical and horizontal fluctuation scales by analyzing cone penetration tests (CPT) carried out in a stiff, overconsolidated clay. The most up-to-date review of the scale of fluctuation in random fields was conducted by Cami et al. (2020). Probabilistic descriptions of soil parameters derived from field and laboratory data and their application in stability analysis were also investigated by Christian et al. (1994), where the first-order second-moment (FOSM) approach was explored and applied to the design of embankment dams. Uzielli et al. (2007) provided a state-of-the-art review of approaches and methodologies for the quantification of soil variability, as well as selected examples of its utilization in reliability-based geotechnical design. In 1997 an original reliability analysis of rigid piles subjected to lateral loadings was proposed by Puła (1997). In the study, he found that random fluctuations of soil properties can cause significant changes in the value of ultimate lateral loading determined according to the Brinch Hansen method. A breakthrough in practical adoption was made in 1997 by Low and Tang (1997), who provided a spreadsheet algorithm to implement the first-order reliability analysis method, which makes reliability analysis much less painful to perform than before. Their spreadsheet algorithm has since found wide applications in geotechnical engineering. Some of the research was published in the ASCE Proceedings “Uncertainty in the Geologic Environment: from Theory to Practice” (Shackelford et al. 1996). The body of work is presented elsewhere (Low 2021).

The last decade in the 20th century brought wider use of probabilistic approaches in geotechnical standards. Some examples of such applications follow here. The first example concerns the implementation of the reliability approach to geotechnical standards in Australia, e.g., Lo et al. (1992) and Li et al. (1993). Becker (1996) proposed incorporating reliability analysis into ultimate limit states of bearing capacity and sliding of shallow and deep foundations in an important study for the National Building Code of Canada. Such an approach aimed to provide a consistent design approach between geotechnical and structural engineers. Another example is a project of Eurocode 7 where reliability methods were not directly used but many elements of these standards were based on them, e.g., Orr and Farell (1999) and Orr (2000). The reliability-based design (RBD), whose goal is to calibrate the resistance (ultimate limit state) or deformation (serviceability limit state) factors in simplified design formats for a selected target reliability index, was also used in the AASHTO LRFD bridge design specifications (AASHTO, 1994). RBD was also applied to foundations for transmission line structures by Phoon et al. (1995, 2003a, 2003b). The application of reliability to calibrate a design guide to achieve an explicit target reliability index was likely first adopted for bridge foundations (Barker et al. 1991). The LRFD calibration approach has since been widely adopted in many AASHTO design problems (e.g., McVay et al. 1998, 2000; Rahman et al. 2002; Paikowsky et al. 2004, 2010; Allen 2005; Allen et al. 2005; Nowak et al. 2007; Zhang and Chu 2009a,b; Abu-Farsakh et al. 2009, 2013; Yang et al. 2010; Abu-Hejleh et al. 2011; Salgado et al. 2011; Smith et al. 2011; AbdelSalam et al. 2012; Ng and Fazia 2012; Penfield et al. 2014; Seo et al. 2015; Motamed et al. 2016; Bathurst et al. 2017; Yu et al. 2017; Haque and Abu-Farsakh 2018; Tang and Phoon 2018; Kalmogo et al. 2019; Ng et al. 2019; Petek et al. 2020). Examples of rigorous reliability theory-based LRFD calibration of internal stability limit states for mechanically stabilized earth (MSE) walls can be found in the papers by Bathurst et al. (2019, 2021) and for soil nails by Lin and Bathurst (2019). Najjar and Gilbert (2009) further examined the effect of a lower-bound capacity in the LRFD design of deep foundations. Phoon et al. (1995) were the first to suggest using multiple resistance factors to accommodate different site conditions. The need to cater for diverse local site conditions and diverse local practices that grew and adapted over the years to suit these conditions was recognized by ISO2394:2015 (International Organisation for Standardization 2015) and the Canadian Highway Bridge Design Code (Canadian Standards Association 2014). A more detailed review of the historical evolution in geotechnical design philosophy and reliability-based design is provided by Phoon et al. (1995). This decade brought the general acceptance of RBD in geotechnical engineering among the geotechnical community.

In the 1990s, along with the wide application of numerical models in geotechnical engineering, Bayesian methods were developed to calibrate the parameters of numerical models (e.g., Reddi and Wu 1991; Ledesma et al. 1996a,b). Honjo and his co-workers developed the extended Bayesian methods for calibrating geotechnical models using observed data from the field (e.g., Honjo et al. 1994; Honjo and Kashiwagi 1999). Gilbert (1999) suggested a first-order second-moment Bayesian method (FOSM) to help calibrate geotechnical models and geotechnical decision making, and the suggested method was later applied for calibrating a numerical flow and transport model based on model test data (Welker and Gilbert 2003). Angulo and Tang (1999) designed an optimal ground-water detection monitoring system using the Bayesian preposterior analysis. Gilbert and Tang (1995) highlighted the challenges in calibrating geotechnical model uncertainty when the model parameters are uncertain and discussed how such uncertainties can be considered using Bayesian methods.

In the 1990s, researchers at NGI contributed to reliability and risk analysis for offshore structures. Lacasse and Goulois (1989) investigated the uncertainty in the API (American Petroleum Institute) parameters for predictions of the axial capacity of driven piles in sand and collated the opinion of 40 international experts in assessing such uncertainties, providing a best-practice example of elicitation of expert judgment. Lacasse and Nadim (1996b) further investigated model uncertainty in pile axial capacity calculations from back-calculations of model tests and comparison of several methods of analyses. Nadim and Lacasse (1992) provided comparative examples of geotechnical stability analyses for offshore structures performed with the effective stress and total stress approaches on a contractant and a dilatant soil. They highlighted the importance of the probabilistic approach, showing that computed failure probability differed significantly for each approach, although the computed factors of safety for the dilatant material were nearly the same. Nadim and Gudmestad (1994) investigated the seismic reliability of a group of offshore platforms to quantify the probability that oil production must be stopped completely given the occurrence of a specific seismic event. Lacasse and Nadim (1994) highlighted the important role of reliability-based approaches and methods in the strengthening of the dialogue between different specialty areas in offshore engineering. Lacasse and Nadim (2007) provided an overview paper on probabilistic geotechnical analyses for offshore facilities, which illustrated a wide range of quantitative methods including event tree analysis, fault tree analysis, Bayesian updating, first-order second-moment (FOSM) method, Monte-Carlo simulations, Bayesian networks, first- and second-order reliability method (FORM and SORM), and system reliability analysis. Nadim and Kvalstad (2007) contributed a keynote paper on risk assessment and management for offshore geohazards. This paper provided a comprehensive set of guidelines and best-practice approaches to identifying, quantifying, and managing geotechnical risks to offshore structures. Cassidy et al. (2015) presented a state-of-the-art review of deterministic and probabilistic advances in the analysis of spudcan foundation behaviour, highlighting the role of Bayesian reasoning in conjunction with the observational method. Researchers at NGI also contributed to the risk assessment of dams. Høeg (1996) contributed a state-of-the-art on the status of risk assessment for dams and proposed a simplified probabilistic risk analysis which was applied in the reevaluation and re-certification of rockfill dams, and to set priority on remedial measures. Vick (1997) summarized risk analysis practice in 11 countries, based on a dedicated survey (e.g., Vick and Stewart 1996).

2000-2010

At the beginning of the 2000s, two Terzaghi Lectures were delivered by researchers from the field of uncertainty quantification in geotechnical engineering. The first was given by Suzanne Lacasse in 2001, titled “Protecting society from landslides - the role of the geotechnical engineer”. The second was given by John T. Christian in 2003, titled “Geotechnical Engineering Reliability: How well do we know what we are doing?”. Baecher and Christian (2003) published an important book on Reliability and Statistics in Geotechnical Engineering, which covers the subject of risk and reliability in both practical and research terms. Vick (2002) published a book on subjective probability and expert elicitation, which is a relatively rare attempt to formalize engineering judgment within reliability and risk analysis.

In the 2000s, the RFEM method became a tool for many geotechnical problems. The use of RFEM for the bearing capacity problems was initiated by Griffiths and Fenton (2001) in their study on the bearing capacity of spatially random undrained clay. The study was later extended to cohesive-frictional soils by Fenton and Griffiths (2003) and the evaluation of slope stability by Fenton and Griffiths (2004) and Griffiths et al. (2009a). RFEM was later used for three-dimensional probabilistic settlement of a foundation by Fenton and Griffiths (2005), where the authors modeled soil as a three-dimensional medium with spatially random Young’s modulus (E) and estimated the reliability of shallow foundations against serviceability-limit-state failure. In 2005, an interesting stochastic approach to the problem of bearing capacity by the method of characteristics was proposed by Przewłócki (2005). The author modified the method of characteristics to consider the randomness of the soil medium in the bearing capacity problem. Jaksa et al. (2005) simulated “virtual sites” using 3D random fields to investigate the effectiveness of site investigation. This virtual site simulation was later extended to grounds with multiple soil layers and lenses by Crisp et al. (2021). The spatial variability impact on three-dimensional long slope failures using RFEM was investigated by Hicks et al. (2008) and Hicks and Spencer (2010). They showed that three failure modes are possible, depending on the ratio of the horizontal scale of fluctuation to the slope size. Griffiths et al. (2009b) showed that ignoring the spatial variability in the third direction as assumed in two dimensional analyses may underestimate the probability of failure of long slopes. One practical outcome of RFEM is the improved definition of a characteristic value. Eurocode 7 (Comité Européen de Normalisation 2004) defines the characteristic value of a geotechnical parameter as a “cautious estimate of the value affecting the occurrence of the limit state.” “The value affecting the occurrence of the limit state” was originally thought to be the same as the classical sample mean for independent data or a spatial average for correlated data defined by Vanmarcke (1977a, 1977b). Hicks and Samy (2002) showed that RFEM is needed to simulate this effective property (called for “reliability-based characteristic value”) for a spatially heterogeneous soil mass. The reason is that the limit state is controlled by a critical failure path that is not the same as an arbitrarily prescribed path. After all, the critical path is identified as the one with the smallest factor of safety. Tabarroki et al. (2020) provided a cheaper but approximate solution to the reliability-based characteristic value. The authors called it the mobilization-based characteristic value. Griffiths and Fenton (2007) and Fenton and Griffiths (2008) published two books on *Probabilistic Methods in Geotechnical Engineering* and *Risk Assessment in Geotechnical Engineering*, which present a thorough examination of the theories and methodologies available for probabilistic modelling and risk assessment in geotechnical engineering, spanning the full range from established single-variable and first-order reliability methods to random field finite element methods.

The 2000s decade brought further applications of reliability approaches to national standards and engineering practice. A good example is Japanese Geocode 21 (Honjo 2005; Honjo et al. 2010; JGS 2006). The reliability approach in geotechnical engineering became more accessible to practitioners (e.g., Duncan 2000) by some algorithm implementations in widely accessible software, e.g., spreadsheet algorithms developed by Low et al. (1998), Low and Tang (2007), Low et al. (2007), and Wang et al. (2010a). Further works on incorporating probability-based methods for liquefaction potential evaluation continued. In the paper by Juang et al. (2002), the authors found that the Bayesian mapping approach is preferred over the logistic regression approach for estimating the site-specific probability of liquefaction.

The 2000s experienced a surge in the application of Bayesian methods in geotechnical engineering. For soil liquefaction potential assessment problems, the Bayesian methods have been widely used to develop new generation liquefaction potential assessment models considering both the aleatory and epistemic uncertainties (e.g., Cetin et al. 2002; Moss et al. 2006). Juang et al. (2002) suggested a Bayesian mapping method to estimate the liquefaction probability based on case histories more realistically. Zhang (2004) illustrated how to use proof pile load tests to verify the reliability of the design of pile foundations. Goh et. al. (2005) used the Bayesian neural network algorithm to model the relationship between the soil undrained shear strength, the effective overburden stress, and the undrained side resistance alpha factor for drilled shafts. Wu (2011) and Wu et al. (2007) illustrated how the Bayesian method can be used as a formal tool to implement the observational method in geotechnical engineering through an embankment construction problem. Other applications are presented in a book edited by Phoon (2008). Zhang et al. (2009) developed a Bayesian framework in which the geotechnical model uncertainty can be calibrated considering the presence of parameter and model uncertainties. The framework also provides the basis for probabilistic back-analysis (Zhang et al. 2010a, b), from which the distributions of several sets of material parameters can be updated. In a deterministic framework, only a limited number of parameters can be back-calculated depending on the number of constraints available. In the study by Yan et al. (2009), the Bayesian probabilistic approach for model class selection was used to revisit the empirical multivariate linear regression formula of the compression index. In 2010, Wang et al. (2010) developed the Bayesian framework in conjunction with cone penetration tests (CPT) to estimate the sand effective friction angle and with random field theory to model the inherent spatial variability. Traditionally, the application of Bayesian methods in geotechnical engineering heavily relied on the conjugate priors where the analytical solution to the posterior distribution is available. Along with the increase of processing power of personal computers, many efficient algorithms have been developed for solving complex Bayesian problems in statistics (e.g., Gelman et al. 2004; Givens and Hoeting 2005). Zhang (2009) reviewed different algorithms for Bayesian computation, clarified the relationships among different algorithms, and assessed the potential of different algorithms for application in geotechnical engineering. Ching and Chen (2007) suggested a Bayesian method to calibrate resistance factors, where Markov chain Monte Carlo (MCMC) simulation is used to solve the posterior distribution. With MCMC simulation, the posterior distribution can be obtained even without the conjugate prior assumption. The MCMC later become one of the main algorithms for solving complex Bayesian problems in the 2010s.

The 2000s is also a decade with wider application of artificial neural networks to geotechnical applications, e.g., Shahin et al. (2001), Shahin et al. (2008), and Jaksa et al. (2008). In the 2000s, the use of more and more sophisticated numerical methods in conjunction with the estimation of uncertainty in geotechnics increased the demand for data from geotechnical surveys and led to the fact that there were more and more of these data, but the possibility of using them was very limited. This problem became even more acute in the next decade. Mitchell and Kopmann (2013) spoke of the availability of big data sets and the challenge to make sense of this wealth of information: "In fact the easy access to such vast amounts of information about most of the topics included in this report, the evaluation of its validity and importance, and deciding which of it should be included was one of the major challenges faced by the authors, and it provided an excellent example of the 'information overload' problem."

It is worthwhile to point out that the Geotechnical Safety Network (GEOSNet) was established in 2006 with the intent to: (1) expand the base of participation in industry and government agencies, (2) raise awareness in practice/education and expedite the transfer of knowledge between research and practice/education, (3) promote sharing of information on the development of geotechnical design codes between countries, (4) promote liaison with related committees within and without the geotechnical engineering community, (5) promote research and practice on assurance of geotechnical safety to keep pace with advancements in numerical methods, risk management methods, materials/equipment, and construction methods, (6) stimulate more discussions on safety issues in complex projects at design, construction, maintenance, and other stages, and (7) encourage engineers to embrace uncertainties and risks more explicitly and more systematically in practice and education. Seven international symposiums (International Symposium on Geotechnical Safety and Reliability; ISGSR) have been organized thus far in Shanghai, Gifu, Munich, Hong Kong, Rotterdam, Denver, and Taipei (https://geosnet.geoengineer.org/).

At around the same time, the first international journal dedicated to geotechnical risk and reliability entitled "Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards" was established in 2007. As pointed out in its first Editorial (Phoon et al. 2007), this journal was motivated by the recognition that "uncertainties associated with geomaterials (soils, rocks, snow), geologic processes and anthropogenic actions can be large and complex. These uncertainties play an important role in the assessment of hazard and risk and in the management of risk" and research needs to be promoted and communicated because "significant theoretical and practical challenges remain for quantifying the uncertainties and developing sustainable risk management methodologies that are attractive to decision-makers and stakeholders." Georisk has since published 15 volumes, 13 special issues, and 5 spotlight articles. It was accepted by SCI in 2020 and received an impact factor of 3.868 in 2021.

2010-present

In the 2010s, researchers at NGI contributed several review/keynote papers regarding geotechnical risk and reliability. In her Rankine Lecture, Lacasse (2015) investigated the conceptual and operational connection between geotechnical practice and risk management and highlighted the importance of implementing concepts of hazard, risk, and reliability in routine analyses. Lacasse (2016) illustrated the use of the reliability and risk concepts with "real life" case studies, specifically for situations encountered in Nordic environments. The paper provided calculation examples from a wide realm of geotechnical problems, including avalanche, railroad safety, mine slopes, and soil investigations. In the Third Suzanne Lacasse Honour Lecture, Nadim (2017) contributed a state-of-the-art paper focusing on concepts such as hazard, exposure, vulnerability, risk, risk management, acceptable risk, and reliability-based geotechnical design. Researchers at NGI also contributed to the risk and reliability of dams. Lacasse et al. (2019) provided an updated overview of basic concepts of reliability-based approaches applied to dams and illustrated their use with three case studies. This recent state-of-the-art paper discussed the strengths of reliability-based analyses and key issues such as tolerable and acceptable risk, the meaning of factor of safety, the targets for margins of safety, and the selection of characteristic values for analysis.

Mechanics-based methods for quantitative risk assessment of slope failure attracted a lot of attention in this decade. Huang et al. (2013) proposed a quantitative risk assessment approach of landslide in spatially variable soils wherein the consequences were assessed individually for each potential failure mode. Zhang et al. (2016) published a monograph titled *Rainfall-Induced Soil Slope Failure: Stability Analysis and Probabilistic Assessment*, which systematically introduced the deterministic and probabilistic methods for analysis of rainfall-induced slope failures as well as their applications in quantitative risk assessment. Wang et al. (2016a) used the random material point method (RMPM) for quantitative risk assessment of slopes where post-failure behaviour can be considered. Their results showed that RMPM provides a much wider range of solutions, in general increasing the volume of material in the failure compared to RFEM solutions. Methods have been developed for assessing the annual failure probability of slopes subjected to seismic shaking (e.g., Wang and Rathje 2018; Huang et al. 2018; Macedo et al. 2018; Zhang et al. 2021), which is essential for risk assessment. Zhang et al. (2021) developed a mechanics-based method to estimate the annual probability of slope failure caused by rainfall infiltration. Moreover, some efficient approaches for three-dimensional bearing capacity estimation for spatially variable soils were proposed, e.g., Chwała (2019) and Li et al. (2021). In the meantime, works on estimating scales of fluctuation remained active, e.g., Lloret-Cabot et al. (2014), Pieczyńska-Kozłowska et al. (2017), Ching et al. (2018), and Cami et al. (2020). The decade brings also some effort to optimization of site investigation programs that maximize robustness and minimize site investigation effort or propose the optimal location of soil soundings (e.g., Li et al. 2016a; Gong et. al 2017; Chwała, 2020; Jiang et al. 2020, Crisp et al. 2020). Recently, some researchers' has turned towards the application of probabilistic methods in conjunction with more complex subsoil models than the Coulomb-Mohr constitutive soil model. These studies used the Hardening-Soil model (e.g., Sert et al. 2016; Luo et al. 2018; Kawa et al. 2021), or the modified Cam Clay model (e.g., Savvides and Papadrakakis 2021).

The 2010s is also a decade with wider application of risk assessment and management combining the reliability and the consequence evaluation in particular with the fields of underground infrastructure system, e.g., tunnels. The International Tunnelling Association ITA (Eskesen et al. 2004) has published a guideline for tunnelling risk management. A notable example of national codes was published in China which is edited by Zhang and Huang (2010). The quantitative risk analysis (QRA) of a tunnel and cut-slope as a system was proposed by Li et al. (2010) considering the failure probability of a risk event occurring in a space domain at a specific time using quantitative vulnerability analysis. Wireless sensor network (WSN)-based risk sensing and monitoring was developed and applied to the real-time risk control in tunnel engineering (Huang et al. 2017). It is followed by a national code of the WSN based risk sensing for infrastructure published in China (Huang 2021). The real time risk sensing along a 20.4km-long metro tunnel lining was applied to the Shanghai metro since 2015. The project is still on-going with at least 6 years continuous monitoring of the operational risk of the metro tunnel (Huang et al. 2017). The next step for risk management of this underground tunnel is to enhance the resilience of the infrastructure (Huang and Zhang, 2016; Zhang et al. 2018).

Due to the continuing increase in computational power, random field geotechnical numerical analyses became increasingly feasible in the 2010s. With the consideration of spatial variability, the failure path and failure mechanisms became more complex and can be correctly treated as unknown prior to the analysis. To address different potential failure modes, system reliability problems are drawing attention in geotechnical reliability and risk, particularly for slope reliability analysis (e.g., Ching et al. 2009; Ching et al. 2010; Huang et al. 2010; Griffiths et al. 2011, Wang et al. 2011; Zhang et al. 2011; Li et al. 2011). Significant challenge encountered in slope system reliability analysis is the prohibitive computational cost when a large number of potential slip surfaces are considered. To address the computational issue, it has been found that the system reliability of a slope is often controlled by a small number of representative slip surfaces (Zhang et al. 2011), and a sub-system comprised of representative slip surfaces can be identified and used to approximate the original and complete system (Zhang et al. 2011; Li et al., 2013). The accuracy of slope system reliability based on representative slip surfaces depends on the correlation of performance functions of different failure modes, which is problem dependent and unknown. This limitation can be alleviated using multiple response surface methods (MRSM) (e.g., Li et al. 2015), where each component performance function is represent by a user-defined response surface. After the response surfaces of all components are constructed, it is trivial to perform system reliability analysis with negligible extra computational cost. Li et al. (2016b) reviewed response surface methods for slope problems. Alternatively, advanced Monte Carlo simulation methods were also developed to tackle computational difficulties in geotechnical system reliability analysis and risk assessment, such as importance sampling (e.g., Ching et al. 2009), subset simulation (e.g., Wang et al. 2011; Li et al. 2016c; Huang et al. 2017), and adaptive Monte Carlo simulation (Liu et al. 2020). These advanced Monte Carlo simulation methods were shown to provide efficient and robust solutions to geotechnical system reliability problems.

The importance of spatial variability, measurement error, statistical uncertainty, and transformation uncertainty for several geotechnical problems in reliability-based design is discussed in 2011 by Honjo (2011). The 2010s decade brought further migration of RBD knowledge and experiences to national standards, e.g., the Canadian Highway Bridge Design Code (Fenton et al. 2016; CAN/CSA-S6-14 2014). In 2015, the 4th edition of the ISO international standard was published, i.e., “General Principles on Reliability for Structures” (ISO 2394:2015), where Annex D was dedicated to the reliability of geotechnical structures (Phoon and Retief 2016). Reviews of semi-probabilistic reliability-based design and direct probability-based design methods are provided by Phoon and Ching (2016) and Wang et al. (2016b), respectively. Cao et al. (2019a) provided a review of Monte Carlo simulation-based methods for full probabilistic design and emphasized values of Monte Carlo samples for geotechnical reliability-based design. In 2014, Juang et al. (2013a) and Juang and Wang (2013) proposed the idea of robust geotechnical design, where the robust design of a braced excavation system (including soil, wall, and support) was formulated as a multi-objective optimization problem, in which the variation of the maximum wall deflection (a signal of the design robustness) and the cost were optimized with the strict safety constraints. In the 2010s further development of opensource software that can be used for uncertainty quantification continued. A good example is OpenCossan software (Patelli et al. 2014). The software was successfully used in a variety of geotechnical applications, e.g., (He et al. 2020). Luo and Bathurst (2018a,b) extended the RFEM to include reinforced slopes and embankments with spatial variability of soil strength. Javankhoshdel et al. (2017) coined the term random limit equilibrium method (RLEM) as an alternative approach to RFEM for probabilistic slope stability analysis of slopes with spatial variability. In the study by Christian and Baecher (2011), ten unresolved problems in geotechnical risk and reliability were identified. Among them was the connection between the observational method and Bayesian updating. In the 2010s, probabilistic back-analysis of slope failure based on Bayesian methods found wide application (i.e., Zhang et al. 2010a, b; Ering and Sivakumar Babu 2016; Jahanfar et al. 2017). The Bayesian method solved with MCMC simulation was also used to implement the observational method for the design and construction of deep excavation problems (e.g., Juang et al. 2013b) and embankments built on soft soils (Zheng et al. 2018). Traditionally, the Bayesian method is often used in geotechnical engineering to back-analyze random variables. Yang et al. (2018, 2019) developed Bayesian methods such that random fields can also be back-analyzed using monitored data. The Bayesian method has been recognized as the main tool for interpreting site investigation data (e.g., Wang et al. 2016c; Juang et al. 2019). It allows for systematic accumulation and updating of site knowledge with increasing data and quantifies site uncertainty to reflect the state of knowledge on site (e.g., Cao et al. 2016). Huang et al. (2018) used Bayesian updating to integrate cone penetration test (CPT) with multi-channel analysis of surface wave (MASW) data for geotechnical site characterisation. It has been noted since the work by Baecher and Rackwitz (1982) that geotechnical data can contain within-site and cross-site variabilities. Extending such an idea, geotechnical data of different groups may also have inter-group and intra-group variability. Recent studies have focused on how to model such variabilities using hierarchical Bayesian models which improve knowledge for one group from data from other groups (e.g., Zhang et al. 2014, 2016; Bozorgzadeh and Bathurst 2020; Ching et al. 2021a, 2021b; Xiao et al. 2021). The Bayesian approach was recently used by Chen et al. (2020) to update knowledge about the design model uncertainties for fixed steel offshore platforms by using data obtained for large soil-structure systems during and after major hurricanes in the Gulf of Mexico.

Possible data-centric future

Bayesian thinking (e.g., Baecher 2017) has been suggested as a foundation for data-driven approaches developed in the 2010s. In 2017, to promote the use of Bayesian methods in geotechnical engineering, the joint working group of TC205 and TC304 published a report to summarize the techniques, advantages, and the application examples of Bayesian methods in geotechnical engineering. Juang and Zhang (2017) provided a practical guide with detailed illustrative examples to learn Bayesian methods in the context of geotechnical engineering. In 2021, Gregory Baecher delivered the Terzaghi Lecture titled “Geotechnical systems, uncertainty, and risk”, where he addressed the importance of the Bayesian approach in geotechnical engineering. Gregory Baecher, Herbert Einstein, Wilson Tang, and TH Wu were among the early Bayesianists in geotechnical engineering. Bayesian methods are now important for machine learning (Phoon and Zhang 2022).

The end of the 2010s decade brought new interest in data itself. Attempts were made to collect characterization of geotechnical data from worldwide surveys and research. Phoon et al. (2019) coined the term Big Indirect Data (BID) to refer to any data that are potentially useful but not directly applicable to the decision at hand. Databases containing property or load test data from multiple sites would belong to BID. Recent reviews on uncertainty representation of geotechnical design parameters and statistical characterization of multivariate geotechnical data were provided by Phoon et al. (2016) and Ching et al. (2016), respectively. Such a database (Project 304 dB, TC304, 2019) is open to the public and is still being updated. Although the main attention was focused on soil properties, significant progress was made in compiling performance data, particularly foundation load test databases (Phoon and Tang 2019; Tang and Phoon 2021). Tang and Phoon (2021) compiled the largest load test database to date, covering various foundation types (shallow foundation, offshore spudcan in layered soils, driven and drilled shaft, and helical pile) in a wide range of ground conditions (clay, silt, sand, gravel and rock) and presented a comprehensive survey of the performance databases for other geo-structures (e.g., soil nail/mechanically stabilized earth walls, slope, plate/anchor, braced excavation).

With the increasing availability of data and the advent of digital transformation, the 2010s decade also brought further usage and development of machine learning approaches in geotechnical reliability and risk analysis, e.g., for slope reliability analysis by Kang et al. (2016), approach for rational and objective interpretation of the soil property profile with quantification of the associated statistical uncertainty (Wang and Zhao, 2017), underground stratification and soil classification (e.g., Cao and Wang, 2013; Depina et al. 2016; Cao et al. 2019b; Wang et al. 2019; Xiao et al. 2021), and landslide susceptibility assessment (Wang et al. 2021a,b). A short review of deep learning is presented by Zhang et al. (2021). Jong et al. (2021) reviewed application of AI techniques to underground soil-structure interaction problems such as characterization of soils and rocks, pile foundations, deep excavations and tunnelling. One major challenge (termed "site recognition challenge") is to quantify "site uniqueness", directly or indirectly, so that big indirect data (BIDs) can be combined with sparse site-specific (local) data in a manner sensitive to site differences (Phoon et al. 2021; Ching et al. 2021a). It is likely that geotechnical data and its role in decision making will be broadened and deepened in the near future in light of rapid advancements in machine learning and artificial intelligence in the past 5 or so years.

Phoon and Ching (2021) and Phoon et al. (2022a) referred to the growing body of research on databases and data-driven methods as "data-centric geotechnics". The central tenet in data-centric geotechnics is that data has value as long as it is not fake. The challenge is to draw useful inferences for decision-making from real world data over the entire lifecycle covering design, construction, operation, maintenance, and decommissioning. Phoon et al. (2021) termed real world data as "ugly data" to contrast with high quality data demanded by the existing deterministic design paradigm. In the 4th Suzanne Lacasse Lecture, Phoon et al. (2019) suggested that the ugly attributes of real world site investigation data can be summarized using the mnemonic MUSIC-X (Multivariate, Uncertain, Unique, Sparse, Incomplete, potentially Corrupted, and spatially variable X). Phoon et al. (2021) pointed that there are other ugly or uglier attributes in rock engineering, because of mixed data types (nominal/ordinal/interval/ratio). The genetic group – sedimentary, igneous, and metamorphic – is an example of nominal data. The joint set number (J_n) in the Q-system is an example of ordinal data. More research is needed to understand data and develop appropriate data-driven methods for rock engineering. The ability to draw useful engineering insights from ugly data ("ugly data challenge") is considered a fundamental problem in data-centric geotechnics. One problem that has attracted attention is site characterization because it is a cornerstone of geotechnical engineering. Ching and Phoon (2019) applied a variant of the powerful Gaussian Process Regression (GPR) method to construct a multivariate probability density function for MUSIC data. This is the first time a probability density function can be constructed in a multivariate setting where the data is both incomplete and sparse, although this setting is commonly encountered in site investigation for a routine project. The Bayesian approach is necessary. This GPR-MUSIC method has since been extended to cover 1D spatial variability (GPR-MUSIC-X, Ching and Phoon 2020) and 3D spatial variability (GPR-MUSIC-3X, Ching et al. 2021c). Wang et al. (2021c) presented a non-parametric approach based on Bayesian compressive sampling. Compressive sampling is distinct from the classical Fourier transform. It can represent a signal with frequencies beyond what is permissible by the sampling interval (Nyquist–Shannon sampling theorem). This is highly advantageous for sparse site-specific data. Shuku et a. (2020) focused on the detection of stratigraphic boundaries using a sparse Bayesian lasso method. The lasso method is by far the best in detecting sudden changes in the sounding profile, which is exactly what is needed for mapping stratigraphy. The key practical purpose of these data-driven methods is to estimate the properties and/or stratigraphic boundaries at unobserved locations based on the typical MUSIC-3X data measured at highly limited locations at a single site. Hu et al. (2021) suggested a method to assess the effectiveness of geotechnical site investigation programs for design of slopes. Phoon et al. (2021) termed this exercise as "data-driven site characterization" (DDSC) and defined DDSC as "any site characterization methodology that relies solely on measured data, both site-specific data collected for the current project and existing data of any type collected from past stages of the same project or past projects at the same site, neighbouring sites, or beyond." Another challenge embedded in DDSC that is distinct from the "ugly data challenge" is the "site recognition challenge". The purpose of this challenge is to improve the estimation at unobserved locations using both site-specific data and data from "similar" sites. The latter data is not sparse as it is assembled from multiple sites (termed "Big Indirect Data" or BID in Phoon et al. 2019). An engineer regularly combines site-specific data with data from other relevant sources through an appreciation of regional geology and experiences gathered from past projects, but there is no data-driven method that can do this comprehensively and satisfactorily to date. The Hierarchical Bayesian model was found to be promising (Ching et al. 2021a; 2021b). Shi and Wang (2021) proposed a novel iterative convolution eXtreme Gradient Boosting model (IC-XGBoost) that can interpolate a subsurface geological cross-section from limited site-specific borehole data and a training geological cross-section obtained from previous projects with similar geological profiles. Phoon and Ching (2021) summarized progress to date on DDSC and postulated that "DDSC may one day evolve into an artificial intelligence (AI) that can emulate human learning and experience building". Phoon (2020) called this longer term project AlphaGeo. Phoon et al. (2021) imagined an artificial intelligence that can mimic human learning to be equivalent to a 'super engineer' that has gained and continues to gain from the pooled experiences of all human engineers around the world. Since it is already known that an experienced engineer makes better judgment than a novice with no experience, one can speculate that such a 'super engineer' with access to extensive databases, capable of detecting site differences through data-driven methods … and capable of moderating its predictions by drawing upon relevant past human experiences will be making better site-specific predictions than those from classical statistical methods moderated by 'reality checks' from a single engineer."

Recognising the potential and growing importance of modern data-driven methods, the journal *Georisk* has expanded its aim and scope to embrace the rapid advancements in machine learning, artificial intelligence, and other data-driven methods and their applications to enhance our design, construction, and decision-making abilities.

To meet the increasing educational need to train professionals with expertise on geotechnical reliability, Zhang et al. (2021) developed a textbook titled "*Geotechnical Reliability Analysis: Theories, Methods, and Algorithms*". A MOOC (Massive Open Online Courses) on *Probability Analysis in Civil Engineering* has been launched by TC304 in the ISSMGE Virtual University to teach reliability theory online.

Remarks

What seems very interesting, in each decade there was a struggle to increase the realism, generality, and numerical efficiency of proposed approaches, regardless of how fast computers we had. It looks like it will stay that way in the future. Nevertheless, with the progress we made, we are capable of solving more and more complex geotechnical reliability problems, making a greater impact in the profession. As Phoon (2020) exhorted in the 10th Lumb Lecture, "the value of geotechnical data is significantly under-appreciated and not fully exploited for decision making. Our data is 'dark' in the sense that it is stored primarily for compliance purposes, rather than shared and actively mined for insights that can inform future decision making. The world is being revolutionised by new and powerful ways of collecting, sharing, analysing, and monetizing data. Clearly, there is a pressing need for the geotechnical engineering community to engage in this digital transformation". The current state of play was discussed in the ISSMGE TC309/TC304/TC222 Third Machine Learning in Geotechnics Dialogue (3MLIGD) (Phoon et al. 2022b). A review of machine learning in geotechnics is presented by Phoon and Zhang (2022).

The ground is complex, because of its natural origin. Important gaps in our knowledge such as geologic history, ground water flow, and possibly recent man-made disturbances from construction or other sources are present in all geotechnical projects. Geotechnical engineering decision making remains a "calculated" risk informed by limited data, incomplete knowledge, and imperfect theories. There is no doubt that insights provided by mechanical, probabilistic, or other analyses need to be moderated by engineering judgment, but it is unclear how engineering judgment should evolve as the capabilities of computational tools and digital technologies continue to grow in power rapidly.

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Time capsule for landslide risk assessment

Lei Yu, Jinsong Huang*, Yifei Cui, Shui-Hua Jiang, Shengnan Wu, and Jianye Ching

Yu Lei
Key Laboratory of Mountain Hazards and Earth Surface Processes
Institute of Mountain Hazards and Environment, Chinese Academy of Sciences, Chengdu 610041, PR China/ China-Pakistan Joint Research Center on Earth Sciences, CAS-HEC, Islamabad 45320, Pakistan
E-mail: leiyu@imde.ac.cn

Jinsong Huang (*Corresponding author)
Discipline of Civil, Surveying and Environmental Engineering
College of Engineering, Science and Environment
The University of Newcastle, Callaghan, NSW 2308, Australia
E-mail: jinsong.huang@newcastle.edu.au

Yifei Cui
State Key Laboratory of Hydroscience and Engineering, Tsinghua University, Beijing 100084, P. R. China.
E-mail: yifeicui@mail.tsinghua.edu.cn

Shui-Hua Jiang
School of Civil Engineering and Architecture, Nanchang University, Xuefu Road 999, Nanchang 330031, P. R. China.
E-mail: sjiangaa@ncu.edu.cn

Shengnan Wu
Center of Emergency Management, Chongqing School of Public Service,

Chongqing 400041, P. R. China.

E-mail: wsn_zju@163.com

Jianye Ching

Dept of Civil Engineering, National Taiwan University, #1 Roosevelt Road Section 4, Taipei 10617, Taiwan.

E-mail: jyching@gmail.com

Abstract

Landslides, one of the most common mountain hazards, can result in enormous casualties and huge economic losses in mountainous regions. In order to address the landslide hazards effectively, the geological society is required not only to develop in-depth understanding of landslide mechanism but also to quantify its associated risk. This article first reviews the history and recent advances of international communities in disaster risk science on the risk models. Landslide risk assessment is categorized into two types, hard and soft risk assessments, and reviewed separately. The hard approach focuses on the mechanics and numerical simulations of individual landslides, subsequent consequences, and their uncertainty quantifications and probabilistic analyses while the soft approach explores the quantification of disaster risk components such as hazard and vulnerability at different scales of concern. It is hoped that this article can serve as a time capsule to link the preceding and following of landslide risk assessments and shed some light to the future studies.

keywords: landslide; mountain hazards; spatial variability; reliability analysis; risk assessment

1. INTRODUCTION

Landslide was defined as the movement of mass of rock, earth or debris down a slope, with the qualification that landslides are not confined to the land nor sliding failure (Cruden 1991). It also includes falls, flows and topples (Varnes, 1978) and has been witnessed on the continental shelf (Twichel et al., 2009) and even on Mars (Quantin et al., 2004). Landslide moves at very different rates from creeping (mm/year) to rapid (10s-100s km/hour) and at different scales from localized slumps (<1 m^3) to a whole mountain collapses (> 10^8 m^3) (Weidinger et al., 2014). At the same time, landslide can be triggered by many factors, such as rainfall, earthquake, snowmelt and anthropogenic activities. Landslides have caused enormous casualties and huge economic losses, especially in mountainous regions. It has been reported that only between 2004 to 2016, a total of 55996 fatalities in nearly 5,000 landslide events were reported (Froude and Petley, 2018) and estimated economic lost at about 3.56 billion USD (EM-DAT). To mitigate landslide hazard effectively, it requires to better understanding of landslide mechanism and its associated risk.

Landslide risk assessment can be categorized into two types: (a) hard risk assessment approach and (b) soft risk assessment approach. The hard approach involves mechanics and numerical simulations of individual landslides, subsequent consequences, and their uncertainty quantifications and probabilistic analyses. In contrast, the soft approach involves conceptual definitions of risk for landslides in a region as well as the subsequent expert-based analyses and statistical analyses. In the latter phase, the soft approach starts to integrate with hard approach by engaging the hazard formation and movement mechanism in the analysis and adopting tools such as numerical simulation in its assessment. Historically, there seems to be little overlap between these two approaches. Therefore, the historical developments for these two approaches are reviewed separately in this document, starting from the soft approach, then the hard approach. For the soft approach, the scope is broadened to "mountain hazards risk assessment" to encompass landslides, debris flows, flash floods, etc. For the hard approach, the scope of the review is limited to soil slope failures.

Recent advances in risk assessment provides systematic processes to improve landslide management. The landslide management has started shifting its focus from landslide control to landslide risk control. During this process, risk assessment has become an essential tool in addressing uncertainty inherent in landslide hazards. Traces the origin concept of risk, this article reviews the major advances in both the soft and hard risk assessment approaches of landslide. Challenges in the future study of landslide risk assessment are also discuss.

2. SOFT APPROACH – MOUNTAIN HAZARDS RISK ASSESSMENT

Mountain hazards refer to natural hazards that occur in mountainous areas which threaten human society, the ecological environment, and natural resources (Cui et al., 2019). This section focuses on such mountain hazards as landslides, debris flows, and flash floods. In the past decade, natural hazards have caused more than 700,000 deaths and more than 1.4 million people injured globally. About 140 million people have been displaced after the disaster. More than 1.5 billion people have been affected by disasters, and economic losses have exceeded 1.3 trillion US dollars (UNISDR, 2015). Among all types of disasters, the widespread and frequent mountain disasters contributed to a significant portion of the casualties and economic losses globally (Cui et al., 2019).

This section firstly chronically arranged the development of mountain hazards risk assessment into four phases as pre-1990, 1990-2005, 2005-2015, and post-2015 according to the milestones for international community engagement in disaster risk reduction. Then, the disaster risk assessment of mountain hazards (e.g., landslide, debris flows, flash floods) was reviewed in terms of its concept, theory, and methodology chronologically in these four phases. In general, although hazard (H) and vulnerability (V) were still two major components, the concept of disaster risk assessment became richer and more comprehensive. At the same time, with science and technology advances, the methods to assess mountain hazard risks and their development at each phase were summarized in this section. In the future study, with the shifting landscape of disaster risks, such as climate change, social and financial crises, vulnerabilities, and social inequalities, the disaster risk assessment should focus more on fundamental research of hazard mechanism in the risk assessment, and also on new forms of the complex, compound, and cascading risks. Meanwhile, with deep roots in natural science, the disaster risk assessment should also integrate social sciences as an evidence-based tool for more solid risk-inform decision-making in disaster risk management.

The idea that risks describe the possibility of loss or injury can be traced to a Greek navigation word in Homer's Rhapsody of *the Odyssey* with the meaning of "cliff or reef" when passing the Sirens, Scylla, and Charybdis during navigation (Skjong, 2005). Later, this Greek word stimulated inspirations for the Latin word *resicum*, *risicum*, and *riscus* with the same meaning of "cliff and reef" and also as a metaphor to describe "the dangers to avoid in the sea" (Skjong, 2005; Boholm, 2015). Subsequently, with the earliest sources dating to the 13th century, many European terms, such as the noun of *risco* in Italian, *risque* in French, *riesgo* in Spanish, took up this Latin word as the point of origins and provided the direct etymology roots for the English word "*risk*" (Collins, 2015; Boholm, 2015).

The concept of risk differs in various fields, including natural sciences, business, health, psychology, insurance, and safety. In the aspect of natural hazards, the United Nations Office for Disaster Risk Reduction (UNDRR) officially defined the disaster risk in 2021 as "The potential loss of life, injury, or destroyed or damaged assets which could occur to a system, society or a community in a specific period of time" (UNDRR, 2021). This section reviews the progress of risk assessment theory and methods related to mountain hazards under the changing concept of disaster risk in the international communities in four phases since the 1990s.

2.1 Milestones for international community engagement in disaster risk reduction

Encountering the unacceptable and rising losses triggered by natural hazards, international communities have taken concerted actions on hazard mitigation and disaster risk reduction with intentions to reduce the impacts from natural hazards since the last century (Fig 1).

The United Nations (UN), as the largest international organization in the world, has funded the United Nations Disaster Relief Office (UNDRO) in 1971, aiming to assist with disaster events in practices, including predisaster planning and technological developments for hazard mitigation (UNDRR, 2017). In 1984, the President of the US National Academy of Sciences, Dr. Frank Press, in the keynote speech at the opening ceremony of the 8th World Conference on Earthquake Engineering, addressed that:

> "*I believe there is great need, and much support can be found, to establish an* ***International Decade of Hazard Reduction****. This special initiative would see all nations joining forces to reduce the consequences of natural hazards*,"

This speech was the earliest inception of the International Decade for Natural Disaster Reduction (IDNDR), although it was narrated as "International Decade of Hazard Reduction." Recognizing the importance of disaster reduction, the UN progressed in the launch of IDNDR and then claimed the 1990s (1990-1999) as the IDNDR. During this decade, the World Conference on Disaster Reduction was held in 1994 and endorsed the Yokohama Strategy and its Plan of Action (1994-2005), which is the first landmark international strategy on disaster reduction which provided guidelines at the international level for risk preparation and prevention and mitigation of disaster impacts. At the end of the IDNDR in 1999, United Nations International Strategy for Disaster Reduction (UNISDR) was founded as the successor arrangements to facilitate the continuous implementation of the IDNDR and later renamed as the United Nations Office for Disaster Risk Reduction (UNDRR). Inheriting the legacy of the Yokohama Strategy to foster international cooperation, UNISDR developed the Hyogo Framework for Action 2005-2015 with the goal to substantially reduce disaster loss in lives, social, economic, and environmental assets by 2015. Subsequently, with the converted concept from "managing disasters" in the Hyogo Framework to the new era of "managing disaster risks", the Sendai Framework for Disaster Risk Reduction 2015-2030 was endorsed in 2015. The Sendai Framework was one of the major agreements in the post-2015 development agenda, working hand-in-hand with the Paris Agreement on Climate Change and the 2030 Agenda for Sustainable Development to provide international communities with concrete actions to protect development gains from the point of disaster risk reduction.

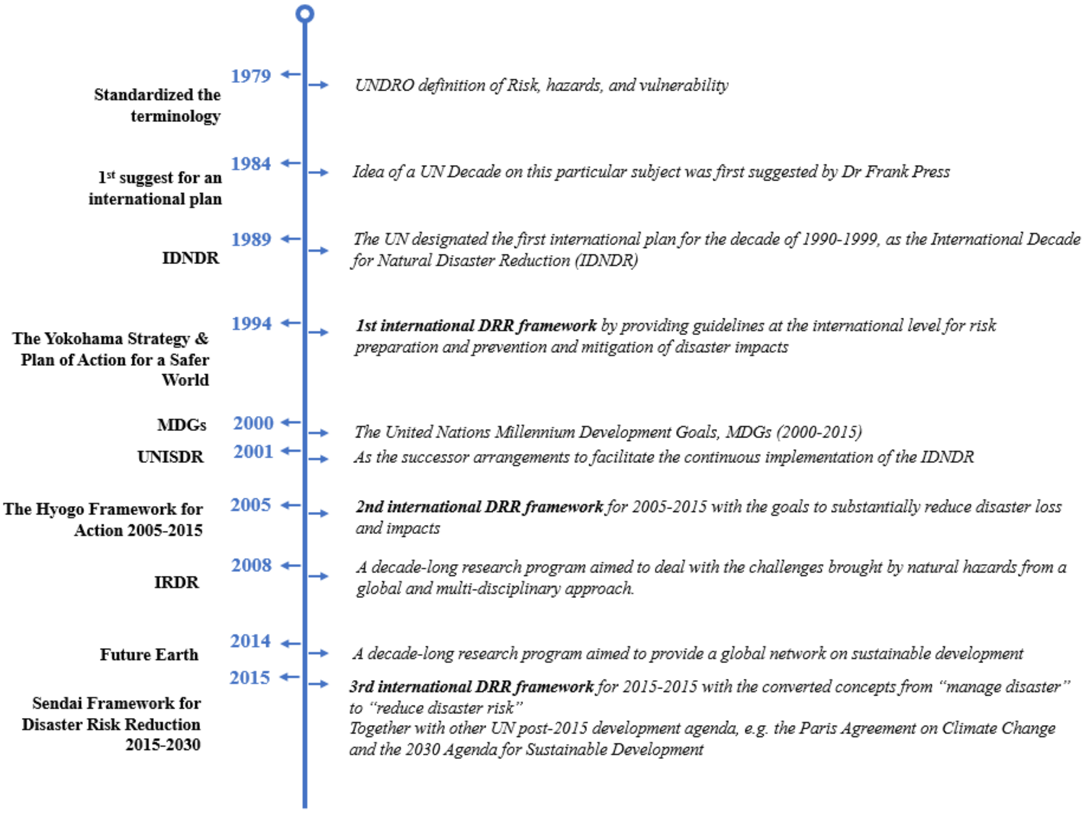

Fig 1 Milestones for international community engagement in disaster risk reduction

In addition to the United Nations, several other influential programs are also devoted to disaster risk reduction. For example, in 2010, a decade-long research program, the Integrated Research on Disaster Risk (IRDR), was established by the co-sponsors of the International Science Council and the UNDRR to deal with the challenges brought by natural hazards, mitigate their impacts, and improve related policy-making mechanisms from a global and multi-disciplinary approach. In 2012, the Future Earth was launched as a 10-year international research program at the UN Conference on Sustainable Development (Rio+20) to provide a global network of scientists, researchers, and innovators collaborating for a more sustainable planet.

At present, international frameworks and research programs have prioritized the understanding of disaster risk in all its dimensions of exposures, vulnerabilities, and hazards for effective and efficient disaster risk management. It is recognized that the role of science and technology in DRR is to develop science-based risk knowledge and methodologies. In this regard, disaster risk assessment is all along provided an essential scientific tool for disaster risk reduction and risk-informed decision-making. The roadmap of disaster risk assessment is reviewed in the following section.

2.2 Phase 1: pre-1990s

In the field of natural hazards prevention, the concept of risk was earliest defined in 1979 in the document "*Natural Disaster and Vulnerability Analysis*" by the Office of the United Nations Disaster Relief Coordinator (UNDRO), a focal point in the UN system for disaster concerns and activities (UNDRO, 1979). Based on relevant publications from UNDRO and UNESCO, this document proposed widely accepted definitions for disaster-related terminologies, for instance, natural hazard, vulnerability, risk, and elements at risk. This glossary provided future disaster risk research with theoretical groundwork in terminology definition. In this document, "risk" was defined as:

"the expected number of lives lost, persons injured, damage to property and disruption of economic activity due to a particular natural phenomenon, and consequently the product of specific risk and elements at risk." (UNDRO, 1979, p.5)

Understanding of disaster risk at this stage emphasizes the losses and potential impact of the disaster. From this point, many researchers had got to a view that "hazard" and "vulnerability" were two crucial components of "disaster risk". The relations among these components of disaster risk are still being explored. One critical feature of disaster risk assessment in this period was to shift the particular emphasis on hazard over vulnerability. Varnes introduced terminologies related to landslide risk in the 1980s, and other scholars proposed minor variations (Varnes, 1984; Einstein, 1988). The assessment methods in this phase, also known as Expert Evaluation Approaches (Leroi 1996), consisted of field geomorphological analysis and overlaying index maps, which rely on human judgment. Application of this method requires the investigators have a thorough knowledge of the causative conditions for the natural hazards. For example, some scholars identified the factors such as precipitation, slope angle, distance to fault, lithology, or vegetation to assess landslide hazards. These factors were numerically evaluated based on their contribution to the hazard and transformed in a spatial format as maps or data banks of points or cells on the ground (Varnes, 1984). These individual factor maps were superposed and integrated to show the hazardous level of the natural hazards. For example, Katsuji Adachi et al. (1977) determined the occurrence probability of debris flow as the basis for debris flow risk assessment from three aspects: geomorphologic conditions, debris flow property, and rainfall. The French ZERMOS procedure (Antoine, 1977) was one of the most comprehensive projects reported in the literature for landslide hazards assessment, and its method was also adopted in areas with similar physical conditions.

Such process was the standardization in assessment procedures from data acquisition to final analysis. However, limited by the development of Geographic Information Systems, the lengthy operations involved in this approach made it less feasible in practice. Stevenson (1977) developed a landslide risk methodology based on hazard zonation in Tasmania. Eldeen et al. (1980) used the hazard map to study the risk of debris flow disaster and introduced the flood risk model to study debris flow risk. Hollingsworth et al. (1981) used the factor superposition method and scoring evaluation to calculate landslide risk assessment, which provided the direction for debris flow risk assessment studies. Hansen et al. (1984) summarized four essential factors for debris flow risk assessment, which laid a theoretical basis for assessment. In 1988, Liu (1988) combined qualitative analysis with quantitative analysis, put forward a multi-factor comprehensive evaluation model of debris flow risk, and proposed a series of risk assessment methods for debris flow of different scales.

2.3 Phase 2: 1990-2005

The 1990s was a remarkable decade for disaster risk reduction. The word "risk society" became a popular discourse (Giddens, 1990; Beck et al., 1992; Mejía-Navarro et al., 1994; Fell, 1994) due to the growing concerns of environmental issues and global development during the period since two sociologists, Ulrich Beck (British) and Anthony Giddens (German), developed the theory of risk society in the 1980s (Caplan, 2000). The United Nations designated the 1990s as the International Decade for Natural Disaster Reduction (IDNDR), during which the United Nations Department of Humanitarian Affairs (UNDHA) in 1992 updated the glossary of agreed terms and specified risk as:

> *"Expected losses (of lives, persons injured, property damaged, and economic activity disrupted) due to a particular hazard for a given area and reference period. Based on mathematical calculations, risk is the product of hazard and vulnerability."* (UNDHA, 1992, p.64)

In this decade, rather than mostly inclination on "hazard" in the previous stage, many researchers also took directions to take account of the "vulnerability" in disaster risk assessment (Dai et al., 2002; Glade, 2003). This growing tendency was closely related to the advent of two major monographs in the field of disaster risk research: "*The environment as hazard*" (Burton et al., 1993) and "*At risk: natural hazards, people's vulnerability and disasters*" (Wisner et al., 1994). These two books provided the fundamental concept of emphasis on vulnerability in disaster risk research. In 1993, Burton published the second edition of the monograph "*The Environment as hazard*" in collaboration with Kates and White (Burton et al., 1993). They systematically analyzed the relationship between development and natural hazards from the perspective of human behavior, emphasizing public choices in the face of disasters. Then, one year later, in 1994, four scientists from different countries, Wisner, Blaikie, Cannon, and Davis, accomplished the foundation monograph of disaster risk research "*At risk: natural hazards, people's vulnerability and disasters"* (Wisner et al., 1994). This book systemically summarizes the relationship between regional development and natural hazards from the perspective of disaster fostering environment, conditioning factors, and elements at risk. It emphasizes the interaction between human vulnerability and hazard factors reflected in the complex risk system.

Henceforth, with additional consideration of vulnerability in risk, many researchers started to explore the relations among risk, hazard, and vulnerability and derived several risk equations (Table 1). Among these equations, the UNDHA (1991) formulated the equation as R= H × V, where disaster risk (R) is a function not only of a hazard (H) but also of the vulnerability (V) of the impact area. This expression of risk had become the fundamental risk equation for much of disaster research in the next two decades, although reserving minor differences in understanding and calculation of H and V, respectively.

Table 1. Concept model of Risk Assessment

Concept model	**Reference**
Risk = Hazard + Vulnerability	Maskrey, 1989
Risk = Probability × Consequences	Einstein, 1988
Risk = Hazard × Vulnerability	UNDHA, 1991; Wisner et al., 1994; UNDP, 2004
Risk = Hazard × Exposure × Vulnerability	Crichton, 1999; Peduzzi et al., 2002; Granger, 2003; Dilley, 2005
Risk = (Σ elements at risk) × Hazard × Vulnerability	Alexander, 2000
Risk = Hazard × Vulnerability × Deficiencies in Preparedness	Villagrán, 2001
Risk = Hazard + Exposure + Vulnerability – Coping Capacity	Hahn, 2003

Among several methods for disaster risk assessment, indicator-based risk assessment took an instrumental role thereupon, primarily when assessing the disaster risk of a large area. The indicator-based approach adopted a similar concept with the Expert Evaluation Approaches in the 1990's. It looked for the disaster fostering and triggering factors, as well as variables that influence the vulnerability of an element at risk. Different sets of indicators could be developed for each type of disaster and elements at risk to reflect their characteristics. The importance of indicators-based method had also been identified in the Hyogo Framework, as a key activity. The development of "systems of indicators of disaster risk and vulnerability at national and sub-national scales will enable policymakers to assess the impact of disasters on social, economic and environmental conditions and disseminate the results to decision-makers, the public and populations at risk" (UN, 2005). In addition, indicator-based risk assessment could overcome the limitations of Expert Evaluation Approaches (such as lengthy procedure and complexity in spatial information processes) with the advances in Geographic Information System (GIS) technology in the 1990s (Knowles, 2008).

The GIS techniques were widely adopted as a tool for spatial data analysis and graphic representation in many fields, including the risk assessment for natural hazards. The GIS-based risk assessment, as a particular interest, was discussed and practiced for many landslides and debris flow assessment (Gupta and Joshi, 1990; Carrara et al., 1991; Mejía-Navarro et al., 1994; Carrara et al., 1995;

Dhakal et al., 1999; Van Westen, 2000; Cavallo and Norese, 2001; Chau et al., 2004; Huabin et al., 2005; Michael-Leiba et al., 2005). Landslide and debris flow assessment relies much on indicator-based methods, where they used environmental factors such as geology, slope gradient and aspect, elevation, soil geotechnical properties, vegetation cover, and triggering factors such as precipitation and earthquake intensity (Mark and Ellen, 1995; Iverson et al, 1998; Guzzetti et al, 1999; Lin et al., 2002; Liu and Lei, 2003; Vallance et al., 2003; Pallas et al., 2004.).

In addition, during this period, GIS techniques also facilitated disaster risk visualization and spatial representation, which exposed risk communication and understanding in a new light to promote practitioners in every industry to gain a deeper understanding and insights hidden in disaster risks. The limitations of this kind of assessment mainly rested in the indicator weight calculation and the lacked hazard mechanism.

2.4 Phase 3: 2005-2015

In this phase, there developed two different approaches to quantify the landslide risk assessment. The first approach relied on the development of probabilistic models. As quantitative risk estimation for geohazards is frequently achieved through the product of two macro-components: hazard and vulnerability of elements at risk. Uncertainty-based risk estimation involves the non-deterministic characterization of all the macro-components of risk, by assigning probability distributions to the hazard and vulnerability or using statistical moments. Lacasse (2007) reviewed the evaluation of slopes safety and realized that the deterministic approach fails to deal consistently with uncertainties. A quantitative framework for landslide risk assessment was then presented where the probabilistic approach, as a complement to the deterministic approach, allowed for hazard and risk assessment of slopes. Uzielli et al. (2009) contributed an uncertainty-based risk estimation where the risk was calculated by Monte Carlo simulation. The non-deterministic characterization of the risk components: hazard and vulnerability, were modeled as functions of intensity, where intensity parameterizes the damaging potential of the hazardous event. Eidsvig et al. (2009) illustrated the application of event tree analysis (ETA) to a rockfall risk analysis at the Aknes site in Norway. The ETA approach is especially useful for geotechnical problems that involve large uncertainties and can quantify hazard and risk, and indicate the most critical situations. Nadim & Liu (2013) proposed a quantitative risk assessment model using Bayesian networks to estimate the risk for the buildings exposed to the threat of earthquake-triggered landslides. The approach included a sensitivity analysis to identify the optimum and appropriate risk reduction strategy in a multi-hazard perspective. Cepeda et al. (2013) proposed a probabilistic procedure based on a Monte Carlo simulation for the probabilistic estimation of landslide run-out extents and intensities in areas where it is not possible to determine the rheological parameters on the basis of back analyses. Liu et al. (2015) proposed a three-level framework for multi-risk assessment where the risk was assessed from qualitative to semi-quantitative and quantitative, based on the impact of the potential hazards. A quantitative multi-risk assessment model based on Bayesian networks is introduced to both estimate the probability of triggering/cascade effects and model the time-variant vulnerability of a system exposed to multiple hazards

Besides using probabilistic models to achieve quantitative risk assessment, with the deepening understanding of the mechanism of mountain hazards such as landslide and debris flow, numerical simulation of disasters became possible (Iverson, 2005; Jakob, 2005; Hungr, 2009). Hazard assessment started to combine indicator-based statistical analysis with numerical simulation. This type of hazard assessment method can depict the mobility and dynamic process of the disaster and the characteristics of debris flow or landslide. Momentum or kinetic energy of the hazard at any location can be calculated from the simulation results and were used for quantitative assessment for the hazard level with higher accuracy and resolution (Hu et al., 2005; Wei et al., 2007; Cui et al., 2015) (Fig 4). In this way, a quantitative and objective connection between hazard intensity and its hazardous level was established.

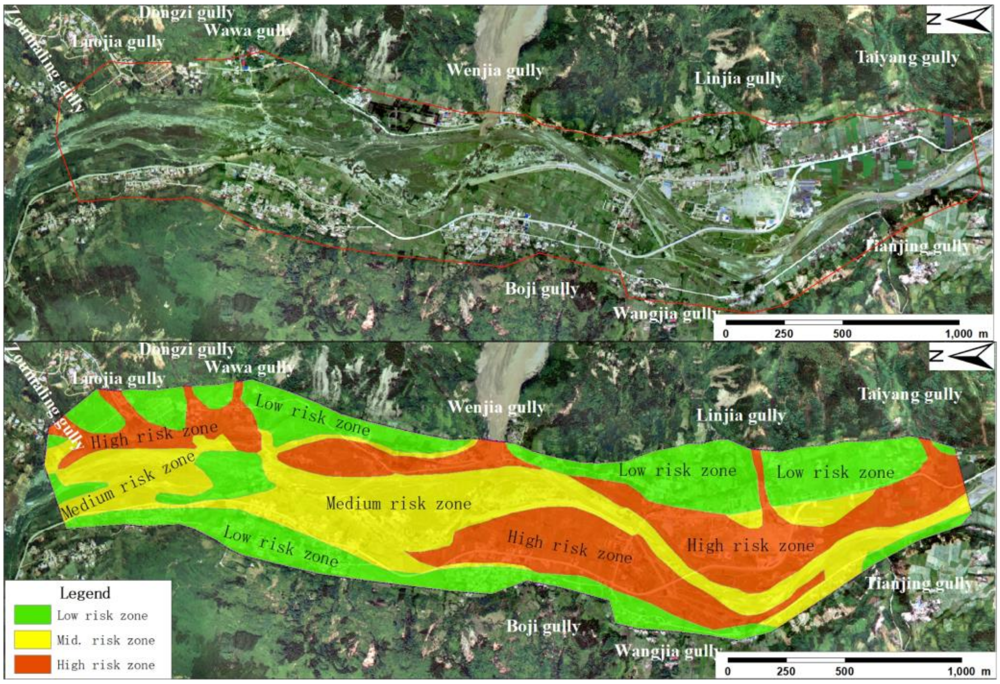

Fig 2. Hazard assessment of Qinping Town (Modified from Cui et al., 2015)

Vulnerability assessment also made significant progress in this period to support disaster risk assessment. In the 1990s, the vulnerability assessment method had transformed from qualitative evaluation or expert judgment to quantitative calculation (Liu et al., 2001). Scholars from Australia and the Alpine countries made an early effort to develop vulnerability matrices to assess the vulnerability of the element at risk based on expert judgment (Leone et al., 1996). Such assessment was qualitative, and the results could be somewhat subjective since the knowledge and experiences of the experts vary. To overcome this limitation, vulnerability matrices were developed in the form of a vulnerability function made as an attempt. Vulnerability functions described the relationship between hazard intensity and the degree of damage for the elements at risk when subject to the hazard impact. Post-disaster survey data were used to correlate the disaster loss to disaster intensity, and the empirical relationship was plotted as vulnerability curve or function using regression analysis (Fuchs et al., 2007; Jacob et al., 2012). Vulnerability curves are popular among practitioners because they directly connect the intensity of a process with the corresponding degree of loss, providing concrete quantitative results and translating potential events into monetary damage. However, this kind of assessment largely relies on the availability of disaster event data such as the debris flow height and monetary damage per building. As such, the developed vulnerability function was very much site-specific and hard to be adopted universally. Uzielli et al. (2008) adopted the ISSMGE Glossary of Risk Assessment Terms definitions on vulnerability as "The degree of loss to a given element or set of elements within the area affected by a hazard…" (http://www.engmath.dal.ca/ tc32/). They proposed a methodology for the quantitative estimation of the physical vulnerability of the built environment and population to landslides. The vulnerability function was defined as the product between landslide intensity and the susceptibility of vulnerable elements, including structures and persons. In addition to vulnerability curves (or functions), more recently, vulnerability indicators were employed to assess the physical vulnerability of buildings and roads (Kappes et al., 2012; Thouret et al., 2014; Zou et al., 2018). Researchers needed to better select indicators, reliable weighting, and aggregation to achieve quantitative assessment (Fuchs et al., 2020).

In this period, advances in understanding hazard mechanisms and well-developed numerical simulation allowed scholars to conduct a dynamic-based hazard assessment. On the other hand, studies on the vulnerability function gave us more objective ways to connect the hazard process to the degree of damage of the element at risk. In this way, site-specific risk assessment transformed from qualitative to quantitative, subjective to objective.

2.5 Phase 4: post-2015

The high level of dependency of modern populations on critical infrastructure and networks allowed the impact of disasters to propagate through socio-economic systems. Where vulnerabilities overlapped and interacted, escalation points were created to create secondary effects more significantly than the primary event (Zhang et al., 2014; Alexander and Pescaroli, 2019). This serial of disaster events was called cascading disasters or disaster chains. Pescaroli and Alexander (2015) defined this type of events by the following statement:

> *"cascading effects are the dynamics present in disasters, in which the impact of a physical event or the development of an initial technological or human failure generates a sequence of events in human subsystems that result in physical, social or economic disruption."*

As people's understanding of risks became more systemic and cascading, risk assessment in this period gradually shifted from the single disaster type to integrated disaster risks. Under this concept, scholars attempted to explore the systemic risks and cascading risks, respectively, from the hazard (H) and vulnerability (V).

Alexander (2018) classified the magnitude scale of cascading disasters into five levels based on the magnitude and potential consequences. This qualitative approach required expert judgment on the escalation point during the chain of events, i.e., where cascades begin and end. At the regional scale, the quantitative assessment method adopted a complex network, Bayesian network, and uncertainty theory to calculate the probability between the adjacent disaster events (Zheng et al., 2017; Han et al., 2019; Guo et al., 2020). These methods could connect the diverse disaster events and related factors to calculate the probability of the occurrence of disaster events so that quantitative hazard assessment of a serial of disaster events could be conducted.

A more quantitative approach was made possible with the numerical simulation on cascading disaster process at the local scale. Shen et al. (2018) and Liu and He (2018, 2020) presented a numerical model to simulate the landslide, barrier lake, and outburst disaster chain, which consists of several sub-models used for simulating the primary and secondary disaster transformation and movement. The numerical model assessed the occurrence of cascading disasters based on the triggering mechanism of each potential disaster in the cascading events instead of calculating the probability of secondary disaster occurrence.

For vulnerability assessment, despite the numerous studies on the physical vulnerability of elements at risk, such as buildings, there was still a gap concerning the interaction between the assessed element and natural process. Scholars had made some attempts by using laboratory tests or theoretical analysis. For instance, Sturm et al. (2018) used a scaled fan model, including building stocks, to capture the flow impact pressure on the walls and provided information on impact pressure and flow heights per building. It demonstrates the importance of shielding effects in buildings vulnerability assessment as some of the buildings acted as protective shields for neighboring buildings, while others may redirect the flow and finally increased the intensity for other buildings. Luo et al. (2020) further assessed the building's vulnerability to mass flows (such as landslides, debris flow, and flood) to investigate the interaction mechanisms between mass flows and buildings using numerical simulation. His study provided a solid basis to develop building vulnerability models for such flow-like mass movements. Based on the recent summary of the pros and cons of the most used vulnerability assessment methods provided by Fuchs (2020), this review paper further included the method of "Vulnerability calculation" in Table 2.

Table 2. Review of existing methods for the assessment of physical vulnerability (Modified from Fuchs et al., 2020)

Method	**Advantage**	**Shortcomings**
Vulnerability matrices	A qualitative method, no need for ex-ante data or detailed information	Results may not be translated into monetary loss. Assessment of damage under specific intensities or process characteristics is objective. No consideration on the property element at risk.
Vulnerability curves	The method is quantitative and may "translate" an event into the monetary cost	Important characteristics of the natural process (e.g., velocity, duration, direction, etc.) and the element at risk (number of floors, construction material) are ignored. Highly demanding in ex-post information. No consideration on the failure mechanism of element at risk.
Vulnerability indicators	Characteristics of the element at risk are taken into consideration	The intensity of the process is not considered, demanding in data (detail, amount quality). No

		consideration on the failure mechanism of element at risk.
Vulnerability calculation	Interaction of hazard process and failure mechanisms of element at risk are considered.	Highly demanding data on the physical property of element at risk (e.g., material strength, dimensions, reinforcement, etc.). Research gaps exist at the calculation of failure process of the vulnerably elements

2.6 **Way forward**

Climate change, biodiversity, social inequality, vulnerability, and other emerging global risk patterns have proposed new eras for disaster risk reduction and its associated issues. In this shifting risk landscape, as an essential tool of scientists in the field of disaster risk reduction, the risk assessment needs to focus on the following aspects.

(1) Climate change, as the new drivers of natural hazards, has posed new challenges on disaster risk reduction and was more complex and changeable. Studies should address climate change associated issues, e.g., extreme events, climate change adaption, and disaster risk assessment, to deepen the understanding of cascading and complex risks and finally form a systematic theory for comprehensive disaster risk assessment.

(2) There should be an emphasis on the fundamental research of disaster mechanisms. Climate changes have brought new characteristic to the nature hazards such as increase in frequency and intensity. Hazard assessment should focus on the mechanism of hazard formation, transformation, and movements, fully address the chaining effect of large-scale hazards. On the other hand, vulnerability analysis needs to focus on the interactions between natural hazards and elements at risk and failure process to achieve more precise quantitative assessment.

(3) Unlike disaster risk assessment with deep roots in natural science, disaster risk reduction has a broader meaning. In addition to natural sciences, it also covers many social sciences such as politics, society, economy, technology, etc. Thus, disaster risk assessment results may become insufficient to support the disaster risk reduction. It is necessary to consider the social and human factors and complete the policy-target risk assessment. The risk assessment method needs to fully assess the needs of policy formulation and policy selection to make better use of the risk assessment results to support scientific decision-making.

3. HARD APPROACH – QUANTITATIVE RISK ASSESSMENT FOR SOIL SLOPES

In contrast to the soft approach of landslide risk assessment, the hard approach involves mechanics and numerical simulations of individual landslides, their uncertainty quantifications and propagations, probabilistic analyses, and subsequent consequences. In the past 50 years, significant advances have been made in quantitative risk assessment of soil slopes accounting for various sources of uncertainties (e.g., spatial variability of geotechnical materials, measurement and model transformation uncertainties, and geological uncertainties) (Jiang et al., 2022). It can be divided into four representative phases, as shown in Fig. 1. Many efficient probabilistic methods have been proposed for modeling of these uncertainties and quantitative risk assessment of soil slopes. The modeling of the uncertainties of soil parameters by a random variable model has been developed to that by a random field model. The slope reliability analysis only considering the inherent uncertainties of soil parameters has been developed to that considering the spatial variability of geotechnical materials, measurement and model transformation uncertainties, and geological uncertainties at the same time. The characterization of the soil spatial variability by an unconditional random field model has been developed to that by a conditional random field model. The slope reliability analysis based on the limit equilibrium method (LEM) with a critical slip surface has been developed to a system reliability analysis involving multiple potential slip surfaces. The initial simple risk assessment of slope failure has been developed into the quantitative risk assessment of slope failures wherein the individual consequence is assessed for each failure mode. The originally simple probabilistic methods, including approximation methods and direct Monte Carlo simulation (MCS) have developed to the advanced simulation methods, auxiliary analysis methods and various surrogate models-based MCS methods.

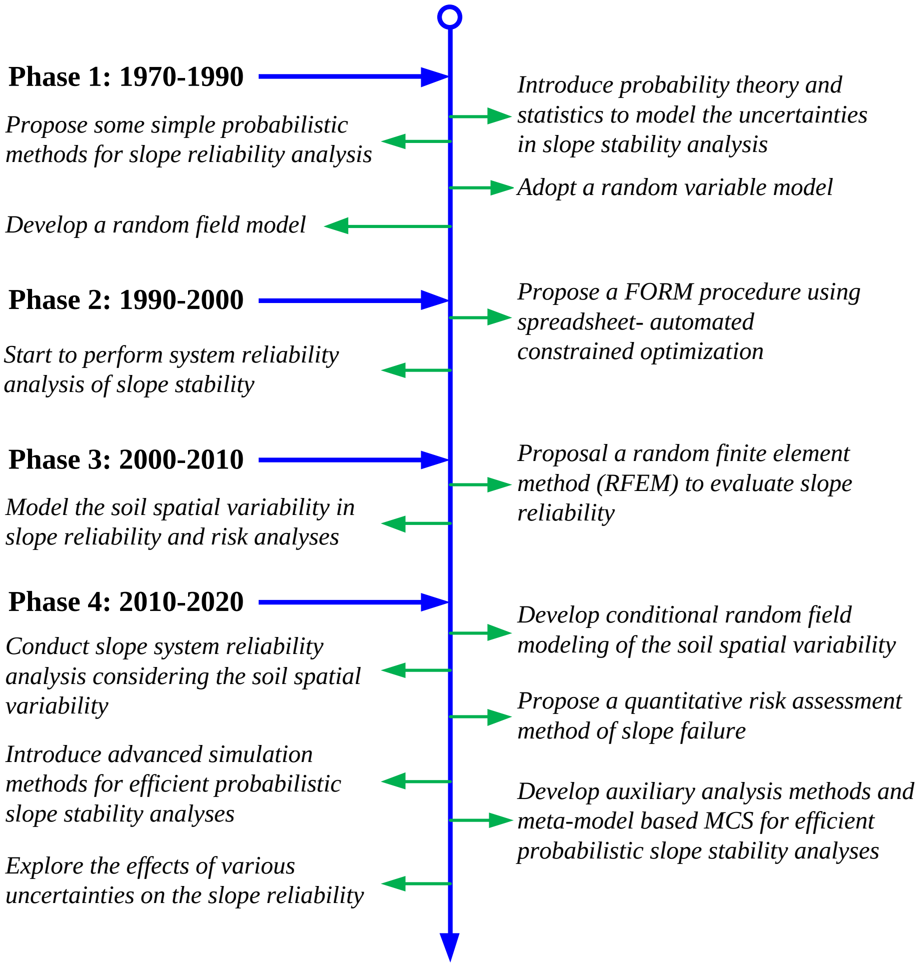

Fig. 1 Evolution history of quantitative risk assessment for soil slopes.

The following subsections aim to give a brief review on the advances in the quantitative risk assessment for soil slopes during the past 50 years. In the first 20 years (1970-1990), probability theory and statistics were introduced to account for the influences of various uncertainties on slope stability. The probability of failure should not be viewed as a replacement, but as a supplement to the factor of safety. In the second decade (1990-2000), the probabilistic methods were greatly developed, a practical and transparent first-order reliability method (FORM) procedure was introduced and system reliability concept was applied in slope stability analysis. In the third decade (2000-2010), research mainly focused on showing the importance of modeling the spatial variability directly in the probabilistic slope stability analysis. In the last decade (2010-2020), a rapid development was observed including quantitative risk assessment of slope failures, and directly using site investigation and field monitoring data.

3.1 Phase 1: 1970-1990

The importance of various uncertainties on the slope stability were realized since the 1970s (e.g., Lumb, 1970; Cornell, 1971; Ang and Tang, 1975; Yucemen and Tang, 1975; Alonso, 1976; Tang, et al., 1976). Then some simple probabilistic methods developed based on the probability theory and statistics were used for the reliability analysis of slope stability. These probabilistic methods can be divided into two categories: approximation methods and direct MCS. The former mainly includes the Hasofer and Lind - Rackwitz and Fiessler (HL-RF) method, point-estimation method (PEM) and first-order second-moment (FOSM) method. The HL-RF method was originally proposed by Hasofer and Lind (1974) and later combined with the Rosenblatt transformation by Rackwitz and Fiessler (1978) to account for the distribution information. In the PEM, the statistical moments of the factor of safety (FS) were approximately estimated without knowing the exact distribution of input random variables (e.g., Rosenblueth 1975, 1981). In the FOSM, the mean and variance of FS were approximated by a first-order Taylor series expansion at the means of uncertain input parameters (e.g., Dolinski, 1982; Li and Lumb, 1987). It is noted that the HL-RF method was the most widely used method due to its simplicity and rapid convergence. Nevertheless, this method may fail to converge under certain conditions (Der Kiureghian and de Stefano, 1991). Some modifications of this method were suggested aiming at the improvement of its robustness. An improved version of this method, termed improved HL-RF (iHL-RF) method, was proposed by Zhang and Der Kiureghian (1995). The direct MCS provided an effective tool for slope reliability analysis due to its simplicity, robustness and flexibility (e.g., Tamimi et al., 1989). To yield accurate results, the direct MCS often requires conducting numerous deterministic slope stability analyses. Thus, the MCS was rarely applied in the slope reliability analysis in this phase because of insufficient computing power.

Random variable model was originally used to characterize the uncertainties of soil parameters for probabilistic slope stability analysis (e.g., Lumb, 1970; Ang and Tang, 1975, 1984). This model treats the soil parameters as random variables, which obey certain probability distributions, such as normal distribution, lognormal distribution, Beta distribution, extreme value I distribution, etc. However, geotechnical materials were formed by a combination of geological, environmental, physical and chemical processes. All soil properties in situ vary vertically and horizontally even within the homogeneous deposits (e.g., Lumb, 1966; Li and Lumb, 1987; Phoon and Kulhawy, 1999). The inherent spatial variability of geotechnical properties has been identified as the most dominant source of the uncertainties. The random variable model that assumes the soil parameters at any point in the studied region are completely correlated with each other, does not account for the inherent spatial variability of soil properties. By contrast, the random field model proposed by Vanmarcke (1977) can be used to explicitly depict the inherent spatial variability of soil properties.

The majority of slope reliability assessments in this phase were performed based on deterministic slope stability analyses using the LEM, including Bishop's simplified method or ordinary method of slices (e.g., Cornell 1971; Tang et al., 1976; McGuffey et al., 1982; Bergado and Anderson 1985; Oka and Wu 1990). Circular failure surface was commonly applied since it is less costly, however, it is appropriate only when the rotational failure mechanism dominates. Non-circular limit equilibrium methods (e.g., Spencer's method) were also used for such a purpose, but convergence problems might be encountered for a non-circular limit equilibrium analysis of slope stability. It is worth noting that the LEM needs to assume the shape and position of the slip surface in advance and cannot account for the stress-strain relationship of the soil.

3.2 Phase 2: 1990-2000

In this phase, the probabilistic methods were greatly developed and widely applied in the slope reliability analysis (e.g., Christian et al., 1994; Low and Tang, 1997; Low et al., 1998; Liang et al., 1999; Malkawi et al, 2000). Of great significance, Low and Tang (1997) proposed a practical and transparent FORM procedure using the spreadsheet-automated constrained optimization based on the perspective of an expanding equivalent dispersion ellipsoid in the original space of the basic random variables. The Rackwitz-Fiessler (1978) equivalent normal transformation was used, but the concepts of coordinate transformation and frame-of-reference rotation were not required. In the FORM, the failure domain is approximated through performing a first-order Taylor series expansion at the so-called design point. The probability of failure (P_f) was then approximated by the probability content under the failure domain defined by the tangent of the limit state function at the design point.

It was found that there might exist multiple dominating failure modes (or slip surfaces) for slope stability problems. These failure modes should be rationally taken into consideration in a system reliability analysis. The system reliability analysis of slope stability attracted increasing interest during this decade (e.g., Oka and Wu, 1990; Chowdhury and Xu, 1995). From the system analysis point of view, the slope was viewed as a series system (Chowdhury and Xu, 1995), in which each potential slip surface is a component and the critical slip surface is the weakest one. Under a probabilistic analysis framework, the overall failure probability (or system probability of failure, $P_{f,s}$) of the slope considering numerous potential slip surfaces is of interest. Some studies were also carried out to locate the probabilistic critical slip surface of a slope (e.g., Hassan and Wolff, 1999). However, the probability of failure of a slope was always larger than that of sliding along any single slip surface due to system effects (e.g., Cornell, 1971; Ditlevsen, 1979; Oka and Wu, 1990; Chowdhury and Xu, 1995). For the slopes with a complex geometry, the probability of failure for the most critical slip surface is typically not the system failure probability of the slope, particularly when the spatial variability of soil properties is considered. To facilitate the slope system reliability analysis, research was conducted to identify the small number of critical slip surfaces among a large number of potential slip surfaces (e.g., Hassan and Wolff, 1999), which are usually treated as representative slip surfaces (RSSs).

3.3 Phase 3: 2000-2010

Although the Vanmarcke's method, FOSM and FORM have been applied for efficient reliability analysis of slopes in spatially variable soils (e.g., El-Ramly et al., 2002; Low, 2007), curse of dimensionality could be encountered when thousands of random variables were involved in modeling the soil spatial variability. These approximation methods were not accurate for tackling the slope reliability problems with a complicated (i.e., highly non-linear) limit state function. A practical consequence was that these methods with constrained circular failure mechanisms superimposed on a random field underestimated the probability of failure of the slope. With the rapid advance in the computing power and technology, the direct MCS and finite element method (FEM) received more and more attention at this phase (e.g., Griffiths and Lane, 1999). A landmark breakthrough is that Griffiths and Fenton (2000, 2004) proposed a random finite element method (RFEM) to evaluate the slope reliability through integrating the FEM with the random field theory in the framework of MCS. A unique advantage of the RFEM was that the program could seek out the weakest (most critical) failure mechanism during each random field realization which might not be a circular shape (e.g., Griffiths and Fenton, 2004; Fenton and Griffiths, 2008; Griffiths et al., 2009a, b). The main challenge for the RFEM was the computational cost, especially when the slope size was very large or the probability of failure was quite small.

The modeling of inherent spatial variability of soil properties in the slope reliability analysis received greater attention at this stage (e.g., Hicks and Samy, 2002; El-Ramly et al., 2002; Cho 2007, 2010). The random field theory was extensively employed to depict the soil spatial variability (Vanmarcke, 2010). For example, Gui et al. (2000) and Srivastava et al. (2000) investigated the influence of the spatial variability of hydraulic conductivity on the slope stability subject to rainfall infiltration. El-Ramly et al. (2002) and Cho (2007, 2010) developed a random limit equilibrium method (RLEM) to assess P_f through integrating the LEM with the random field theory in the framework of MCS. Griffiths and Fenton (2004) and Griffiths et al. (2009a) found that ignoring the spatial variability of soil shear strength parameters would overestimate the P_f of a slope when the coefficients of variation of shear strength parameters were relatively high or the FS was relatively low. Griffiths et al. (2009b) showed that ignoring the spatial variability in the third direction as assumed in two dimensional analyses could underestimate the probability of failure of long slopes.

3.4 Phase 4: 2010-2020

At this phase, random field discretization methods, and efficient probabilistic and risk analysis methods for the spatially variable slopes were developed rapidly. Unconditional random field (URF) model was widely used in the early stage for the characterization of the soil spatial variability, but it cannot incorporate the site-specific data to constrain the spatial distribution of soil parameters. The URF simulations generally overestimated the random field variance in a conservative design scheme because it did not make full use of real data at a specific site (e.g., Rubin et al., 2010). To this end, various conditional simulation methods including geostatistics approach (i.e., ordinary Kriging, cokriging, sequential Gaussian simulation), Hoffman method, Bayesian and Sobol' index methods were developed (e.g., Ching et al., 2012; Li et al., 2016a; Liu et al., 2017; Lo and Leung, 2017; Jiang et al, 2018;

Mouyeaux et al., 2018; Johari and Gholampour, 2018; Huang et al., 2019; Johari and Fooladi, 2020). These methods laid a solid basis for rational reliability and risk assessments of soil slopes by making optimal use of site-specific data.

Most of previous studies focused on various slope failure modes caused by stratification (i.e., layered soils), while those caused by the inherent spatial variability of soil properties in each soil layer were rarely investigated. To incorporate the inherent spatial variability into system reliability analysis of slope stability, Huang et al. (2010) applied the RFEM to evaluate the system reliability of slope stability in spatially variable soils. It is straightforward to use the RFEM for system reliability analysis of slopes because no prior assumption is made for slip surfaces in the RFEM. However, it becomes tedious to use the RLEM for system reliability analysis of slopes because prior assumptions about slip surfaces are required in the LEM. To improve the efficiency of RLEM, an improved strategy is to identify a limited number of RSSs that constitute a slope subsystem having the same $P_{f,s}$ as the slope system comprised of all the potential slip surfaces (e.g., Zhang et al., 2011; 2013; Jiang et al., 2015). For instance, Jiang et al. (2015) developed an efficient system reliability analysis approach based on the direct MCS and LEM to estimate $P_{f,s}$ in spatially variable soils. These approaches allowed explicit modeling of the inherent spatial variability and improved the computational efficiency of slope system reliability analysis using the RSSs and surrogate models. The above studies deemed that a slope subsystem were constituted by the RSSs with a large, but limited, number of circular slip surfaces. This is not rational for the slope with non-circular slip surfaces wherein the spatial variability of soil properties and weak seams are involved. As such, Liu et al. (2020) proposed an adaptive MCS method for system reliability analysis of slope stability based on the LEM, which allowed the incorporation of a large number of circular and/or non-circular slip surfaces.

The quantitative risk assessment of slope failures also attracted a lot of attentions in this decade (e.g., Ali et al., 2014; Li et al., 2016b; Zhang and Huang, 2016; Cheng et al., 2018; Li et al., 2019a; Ng et al., 2021). This was owing to the fact that Huang et al. (2013) proposed a quantitative risk assessment approach of landslide in spatially variable soils wherein the consequences were assessed individually for each potential failure mode. The traditional method didn’t work well for the systems that had many possible failure modes wherein the spatial variability was considered (e.g., Huang et al., 2010; Jiang et al., 2015). Each of possible failure modes has individual consequence since the sliding volumes underlying different failure modes are distinctly different (e.g., Zhu et al., 2015; Li et al., 2019a). Currently, to pursue a rational quantitative risk assessment of slope failures, the random material point method (RMPM) and random smoothed particle hydrodynamics (RSPH) have been developed to model the post-failure behavior, and to reasonably estimate the probability of slope failure, influence zone, runout distance and landslide consequence (e.g., Wang et al., 2016, 2019; Liu et al., 2019; Mori et al., 2020; Zhang et al., 2020; Liu and Wang, 2021; Liu et al., 2021).

In the meantime, many advanced simulation methods such as subset simulation, importance sampling, Latin hypercube sampling and line sampling have been introduced to promote the probabilistic slope stability analyses (e.g., Ching et al., 2009; Au et al., 2010; Wang et al., 2011; Tang et al., 2020; Liu et al., 2020). For example, Wang et al. (2011) used an advanced MCS method called "subset simulation" to calculate P_f. Jiang and Huang (2016), Huang et al. (2017) and van den Eijnden and Hicks (2017) developed subset simulation-based methods for estimating the small probability of failure considering the spatial variability of soil properties. Besides, auxiliary analysis methods were developed through making optimal use of the merits of LEM and FEM, i.e., the computational efficiency of LEM and the FEM's capability of giving a more realistic prediction of slope stability (e.g., Li et al, 2016c; Xiao et al, 2016; Song et al, 2021). Moreover, Liu et al. (2019) proposed a RLE-MPM by taking advantage of the merits of LEM and MPM, i.e., the computational efficiency of LEM and the MPM's capability of modeling the post-failure behavior of a landslide considering the soil spatial variability. Liu and Wang (2021) further developed a RFE-MPM by utilizing the merits of FEM and MPM, i.e., the computational efficiency of FEM in hydro-mechanically coupled analysis before landslide initiation, and the capability of MPM in simulating the post-failure large deformation of rainfall-induced landslides. These auxiliary analysis methods offered a new insight for the quantitative risk assessment of slope failures in spatially variable soils.

To reduce the computational cost of the direct MCS, a surrogate model-based method which requires much less evaluations of slope stability was also developed (e.g., Li et al., 2016d; Zhu et al., 2019; Wang and Goh, 2021; Zhou et al., 2021; Deng et al., 2021). The polynomial chaos expansion, Kriging, support vector machine, Gaussian process regression, multivariate adaptive regression splines and convolutional neural network were commonly adopted to construct the surrogate models of the factor of safety. The direct MCS, subset simulation or other simple probabilistic methods were then employed to estimate the P_f. For example, Jiang et al. (2015) proposed a Hermite polynomial chaos expansion-based MCS method to calculate the P_f in spatially variable soils. Zhu et al (2019) utilized a Gauss process regression-based MCS method to calculate the P_f in spatially variable soils. He et al (2020) and Wang and Goh (2021) calculated the P_f of a spatially variable slope using a deep learning-based MCS methods. Deng et al (2021) developed a multivariate adaptive regression splines-based MCS method for the slope reliability analysis in spatially variable soils.

Furthermore, the influence of stratigraphic uncertainty on the slope reliability and quantitative risk analyses was also widely accounted for in the phase (e.g., Deng et al., 2017; Wang et al., 2018; Gong et al., 2019; Juang et al., 2019). The coupled Markov chain (e.g., Qi et al., 2016; Deng et al., 2017; Li et al., 2019b; Deng et al., 2020), stochastic Markov random field (e.g., Li et al., 2016e; Wang et al., 2018; Wang, 2020), and random field (e.g., Gong et al., 2020; Zhao et al., 2021) were introduced to characterize the geotechnical and geological uncertainties at the same time.

3.5 Way forward

There are still many challenges and research possibilities for the hard risk assessment of soil slope failures, which may include, but not limited to the followings:

(1) The available field data is generally Multivariate, Uncertain and Unique, Sparse, Incomplete, and potentially Corrupted with "3X" denoting three-dimensional (3D) spatial/temporal variability (MUSIC-3X) as stated by Phoon et al. (2021). How to accurately characterize site characteristics of soil properties based on the field data with "MUSIC-3X" in the hard risk assessment remains an open

question and has not been systematically explored.

(2) It is fairly computationally expensive for the risk assessment of large three-dimensional slope failures at small probability levels wherein the entire slope failure process should be modeled. Thus, the ability to account for the progressive failure of the slope will be one of the important directions for the quantitative risk assessment of landslides.

(3) To accurately assess the true consequences of slope failure (the extent of failure), the random material point method and random smoothed particle hydrodynamics method has been gradually applied to model the progressive failure of the slope, which will become one of the important directions in the quantitative risk assessment of landslides.

4. REMARKS

Mountain regions cover approximately 24 percent of the Earth's surface. Landslide, as one of the most common hazards, is widely distributed in this region and threats about 13 percent of the world's population inhabits them. We have come a long way and developed various approaches so that we could assess and understand the existing risk more thoroughly. However, the "Global assessment report on disaster risk reduction 2019" (UNDRR, 2019) indicated that 'At no point in human history have we faced such an array of both familiar and unfamiliar risks, interacting in a hyperconnected, rapidly changing world. New risks and correlations are emerging. Decades-old projections about climate change have come true much sooner than expected. With that come changes in the intensity and frequency of hazards. Risk really is systemic, and requires concerted and urgent effort to reduce it in integrated and innovative ways.' Understanding the risk creation and perpetuation in the contemporary risk landscape will bring us new challenges but at the same time the opportunities to work with all disciplinaries that required to achieve more comprehensive landslide risk assessment.

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